THERMAL SIMULATION OF LOUDSPEAKERS

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1. Abstract

The paper describes the development of a system to simulate accurately the temperatures of the voice coil and magnet assembles in moving coil loudspeakers in real-time for any programme material.

After initial parameter measurements on the loudspeaker there is no further need for connection to the loudspeaker, the temperatures can be simulated independently. The simulation has been verified through measurements performed on bass, midrange, and tweeter units.

2. Introduction

A great problem for loudspeaker designers and drive unit manufacturers is loudspeaker burn-out. This is caused by the heating effect of the voice coil when a signal is applied to it. A point is reached where either the resin on the voice coil melts causing short circuits or the flux of the magnet structure is damaged causing loss of sensitivity and control. A 10 % loss in flux can occur for temperature changes of ~90 degrees Celsius (°C) for neodymium magnets [1]. Failure of the unit results. The temperatures at which these phenomena occur depends on the drive unit, for some units, temperatures exceeding 250 °C close to the voice coil can be generated.

Another problem associated with this heating effect is thermal compression. Due to the increase in temperature, the impedance of the drive unit also increases causing a lower power to be applied to the loudspeaker resulting in a lower acoustic output. Also, the electrical Q of the drive unit is dependent on the DC resistance of the voice coil, which changes with temperature, thus effecting the transfer function of the whole system.

To solve the problems mentioned above we need a knowledge of the actual voice coil temperature that would occur during operation. Some systems have been developed to calculate the temperature of the voice coil during operation of the loudspeaker system. Dr. Ing Gottfried Behler [2] created a system that measured the loudspeaker's impedance during operation as a way to extract the DC resistance of the coil and hence extract the temperature. The author quotes an error of less than 5 °C.

However, a system that could give temperature data for a loudspeaker system without a need for the loudspeaker to be under operation would be advantageous and allow flexible simulations to be made at any time, with any programme, without the risk of damaging any units.

3. Our Requirements For A Thermal Simulation System

The system should be capable of,

- receiving a standard digital audio signal from Compact Disc (CD) in S/P DIF format.
- filtering the input signal with a high order crossover.
- introducing an amplifier by means of a voltage gain and clipping function.
- calculating accurately the temperatures of the voice coil and magnet assemblies.
- running in real-time for up to a four-way, stereo, loudspeaker system without connection to the loudspeaker.

4. Thermal Modelling A Moving Coil Loudspeaker - A New Model

The first consideration is modelling the thermal behaviour of the loudspeaker drive unit. This section describes the development of an accurate thermal model, a model that is different to the model used previously by Henricksen [3] and other authors [1,4], a defence of which is given.

Previous work in this area has led to a model that is widely accepted and has been used for the derivation of temperatures. The model is shown in figure 1 and is a lumped-element model consisting of two cascaded parallel RC networks. The current flowing into the model is the power, P, applied to the drive unit and the two RC time constants (R_1C_1, R_2C_2) represent the thermal time constants of the voice coil and magnet systems respectively. T_a represents the surrounding air temperature or ambient temperature and the voltages T_{vc} and T_m represent the temperatures of the voice coil and magnet as lumped elements, above ambient. The s-plane transfer functions (were s = jω) for this model are,

$$T_{vc} = P\frac{R_1 + R_2 + s(R_1R_2C_2 + R_1R_2C_1)}{1 + s(R_1C_1 + R_2C_2) + s^2(R_1C_1R_2C_2)} \qquad \textbf{Eqn 1}$$

$$T_m = P\frac{R_2}{1 + s(R_2C_2)} \qquad \textbf{Eqn 2}$$

Consider now a new model, shown in figure 2. The current flowing into the model is again the power, P, applied to the drive unit and the time constants R_1C_1 and R_2C_2 represent the thermal time constants of the voice coil and magnet systems respectively. T_a represents the surrounding air temperature or ambient temperature. The voice coil temperature, T_{vc}, is now given by the potential across the thermal capacitance C_1 and the magnet temperature, T_m, by the potential across the capacitor C_2. The s-plane transfer functions for this model are,

$$T_{vc} = P\frac{R_1 + R_2 + s(R_1R_2C_2)}{1 + s(R_1C_1 + R_2C_1 + R_2C_2) + s^2(R_1C_1R_2C_2)} \qquad \textbf{Eqn 3}$$

$$T_m = P\frac{R_2}{1 + s(R_1C_1 + R_2C_1 + R_2C_2) + s^2(R_1C_1R_2C_2)} \qquad \textbf{Eqn 4}$$

By comparing equations 1 and 3, it is clear that the two models can have the same input impedance and are therefore both equally valid for the calculation of the voice coil temperature. However, it is not possible to obtain equal magnet temperature functions.

4.1 Second Order Thermal Model Step Responses

Consider now the step response of the new model. The instant a step is applied, power P flows into the voice coil and energy is dissipated in the coil. The remaining energy, P_2, is fed through to the magnet system. Initially the powers will be different but will become equal when the current flow through C_1 has stopped i.e. thermal conditions in the voice coil have stabilised.

Applying a step input to the model in figure 1, it is seen that equal power, P, must flow into both the voice coil and magnet systems. Hence, there can be no initial settling of the voice coil system to effect the magnet temperature. Figures 3 and 4 show the voice coil and magnet temperatures, respectively, for the first few seconds when a step is applied to each model. The component values used are the actual thermal component values for a 19 mm tweeter. The component values for the model in figure 2 are (rounded to 3 s.f.),

R_1	=	9.25	C_1	=	0.0910
R_2	=	35.1	C_2	=	15.2

The values for the model in figure 1 have then been adjusted to give the same voice coil temperature for the first 40 seconds, within 2×10^{-7} °C. The values are (rounded to 3 s.f.),

R_1	=	9.14	C_1	=	0.0916
R_2	=	35.2	C_2	=	15.2

The time constant of the voice coil (R_1C_1 = 0.84 s) and its effect on the magnet temperature, in figure 4, given by the new model (solid curve) can be clearly seen. However, the magnet temperature given by model shown in figure 1 (dotted curve) does not account for the presence of the voice coil. Therefore, the time constant of the voice coil has no effect on the magnet temperature.

One may argue that the difference between the two models is small, which is true in this example of a tweeter with a very short voice coil time constant, but the difference will be exaggerated for voice coils with much longer time constants and for signals other than step functions, for example music.

For the above reasons and due to the interest in simulating both the voice coil temperature and magnet temperatures, the new model in figure 2 was initially chosen to model a moving coil loudspeaker.

4.2 A New Third Order Thermal Model

After development of the new second order model it was found that two time constants was not sufficient to model the thermal behaviour accurately. A third order model was then developed and used for the simulation system. This model is shown in figure 5.

Figures 7,8 and 9 reveal the limitations of the second order model. They show the measured and modelled voice coil temperature step responses for a 170 mm woofer. The measured response is the same in all three figures and was measured as described in section 4.3.1. Figure 7 shows the early part of the step response.

It is clear that with two time constants it is not possible to model the complete step response of the system. However, three time constants allow the complete thermal behaviour of the drive unit to be modelled. The modelled thermal component values for the second order model are (to 4 s.f.),

R_1	=	4.936	C_1	=	2.962	(R_1C_1	=	14.62 s)
R_2	=	2.843	C_2	=	741.4	(R_2C_2	=	2108 s)

and the third order model,

R_1	=	3.927	C_1	=	2.358	(R_1C_1	=	9.260 s)
R_2	=	1.158	C_2	=	71.49	(R_2C_2	=	82.79 s)
R_3	=	2.634	C_3	=	664.3	(R_3C_3	=	1750 s)

It is seen that the shortest and longest time constants in each model correspond and that the third order model introduces another time constant relatively close to that of the voice coil. Therefore, the temperatures given by the third order model, indicated in figure 5, are the voice coil temperature T_{vc}, the magnet temperature T_m and a third temperature T_g relating to the gap temperature or the temperature of the magnet surface close to the voice coil, above ambient.

Thoughts that the longest time constant in the third order model relates to the cabinet can be quickly expelled by monitoring the ambient temperature within the cabinet during measurement of the step response. Also, for measurements and simulations with a music signal it is seen how the voice coil temperature quickly cools to the surrounding magnet temperature and in the case of a tweeter it is seen how the magnet temperature (as calculated with the third order model) fluctuates which is clearly not the cabinet temperature (figure 28).

The transfer functions for the third order model in figure 5 are as follows. The voice coil temperature above ambient,

$$T_{vc} = P\frac{R_1+R_2+R_3+s(R_1R_2C_2+R_1R_3C_2+R_1R_3C_3+R_2R_3C_3)+s^2(R_1R_2R_3C_2C_3)}{\begin{array}{c}1+s(R_2C_2+R_3C_2+R_3C_3+R_1C_1+R_2C_1+R_3C_1)+s^2(R_2R_3C_2C_3+R_1R_2C_1C_2\\+R_1R_3C_1C_2+R_1R_3C_1C_3+R_2R_3C_1C_3)+s^3(R_1R_2R_3C_1C_2C_3)\end{array}}$$

Eqn 5

The gap temperature,

$$T_g = P\frac{R_2+R_3+s(R_2R_3C_3)}{\begin{array}{c}1+s(R_2C_2+R_3C_2+R_3C_3+R_1C_1+R_2C_1+R_3C_1)+s^2(R_2R_3C_2C_3+R_1R_2C_1C_2\\+R_1R_3C_1C_2+R_1R_3C_1C_3+R_2R_3C_1C_3)+s^3(R_1R_2R_3C_1C_2C_3)\end{array}}$$

Eqn 6

The magnet temperature,

$$T_m = P\frac{R_3}{\begin{array}{c}1+s(R_2C_2+R_3C_2+R_3C_3+R_1C_1+R_2C_1+R_3C_1)+s^2(R_2R_3C_2C_3+R_1R_2C_1C_2\\+R_1R_3C_1C_2+R_1R_3C_1C_3+R_2R_3C_1C_3)+s^3(R_1R_2R_3C_1C_2C_3)\end{array}}$$

Eqn 7

The voice coil step response of the third order model is of the form,

$$H_{vc}(t) = A_0 + A_1 e^{a_1 t} + A_2 e^{a_2 t} + A_3 e^{a_3 t} \qquad \textbf{Eqn 8}$$

Where the coefficients A_n and a_n are related to the thermal component values R_n and C_n and the power P by multiplying equation 5 by 1/s and performing the inverse Laplace transform. The derivation is lengthy, requiring the use of partial fractions and the solving of third order polynomials. Hence, the derivation has not been reproduced here.

4.3 Measurement Of The Thermal Component Values

We have seen that the third order thermal model can accurately describe the thermal behaviour of a moving coil loudspeaker. It now remains for the component values, R_n and C_n, in the model to be determined for any drive units of interest. With these parameters it is then possible to use the model in a simulation to obtain the temperatures of the voice coil etc. from a knowledge of the power, P, applied to the loudspeaker unit.

The thermal parameters of the model, R_n and C_n, are derived from a measurement of the step response of the loudspeaker. This step response is the rise in voice coil temperature when a constant voltage is applied to the drive unit.

A constant voltage (or step) is applied in the form of a constant amplitude sinewave at a frequency where the impedance phase shift of the loudspeaker unit in the cabinet is zero. At this frequency the loudspeaker presents a resistive load and the precise power dissipated is then known. With an applied rms voltage v, the current flowing in the voice coil is given by,

$$i = \frac{v}{(R_e + R_m)} \qquad \textbf{Eqn 9}$$

Where R_e is the initial DC resistance of the voice coil and R_e+R_m is the initial magnitude of the impedance at the chosen frequency of zero phase shift. The power, P, is then given by,

$$P = i^2 R_e \qquad \textbf{Eqn 10}$$

In practice R_e is a function of temperature, given by equation 14 (section 5.5). The voice coil temperature is then measured while the step is applied to the loudspeaker. The voltage is chosen such that the drive unit is not damaged during the measurement which has to be sufficiently long to allow determination of all three time constants of the third order model. Measurement of the voice coil temperature and determination of the thermal parameters for three drive units is now described.

4.3.1 Measuring The Voice Coil Temperature

For the purpose of calculating the thermal component values and for verifying the simulation, the voice coil temperature was measured using two methods. Firstly, by measuring the change in DC resistance of the voice coil where a small DC current flow in the coil allows the temperature of the coil to be calculated from the increase in voltage, and secondly, in the special case of a 170 mm woofer, using a thermocouple wound into the voice coil. This woofer was specially constructed by Peerless Fabrikkerne A/S of Denmark.

In each case, the voltage measured (either due to the change in DC resistance of the coil or from the thermocouple) is recorded using an 8 bit datalogger inside a personal computer. This gives a measurement resolution of ~0.5 °C for measurements using the change in DC resistance of the coil (the precise resolution is given for the particular drive unit) and a resolution of 0.78 °C for measurements using the thermocouple. The datalogger samples data at a frequency of 10 kHz and exports an average at intervals specified, for example each second.

In practice, two measurements of the step response are made with the same voltage and with the system at the same initial temperature equal to the ambient temperature (which is monitored during measurements to ensure there are no changes to influence the results). The two measurements are a long measurement and a much shorter measurement, the duration of which are determined such that all time constants fall well within the long measuring time. The datalogger exports data at a much higher rate in the short measurement. This allows accurate derivation of all three time constants which are calculated from a curve fitting algorithm that fits the model (described by equation 8) to the measurements. Details of the measurements are given below for three difference drive units.

4.3.2 Measurement Of The Loudspeaker's Impedance

For the step response it is necessary to apply a sinewave to the drive unit at a frequency where the impedance of the unit in the cabinet has zero phase shift. Therefore, it is necessary to measure the impedance of the driver in the cabinet. Measurements for this study have been made using a Bruel & Kjær Audio Analyser type 2012. A frequency can then be selected from the measurement where the phase is zero. Note, however, that the phase will shift through zero several times (twice for a driver in a sealed cabinet). The higher of the frequencies should be selected as the gradient of the phase shift is lower and therefore minimises errors caused by slight variations in the frequency for the step response, also the oscillations of the sinewave will not be transposed onto the measurement as the frequency is not close to DC. The temperature at which the impedance measurement is made should also be recorded and the step response measurements made at the same temperature, thus the DC resistance R_e and the value $R_e + R_m$ from the impedance measurement can be used for direct calculation of the power applied during the step response. The impedance curve is also required for calculation of the power in the simulation as described in section 5.5.

4.3.3 Thermal Components For A 170 mm Woofer

The woofer was placed in a 50 litre sealed cabinet to give a smooth bass response with a -3 dB frequency of 40 Hz. For measurement of the thermal step response a sinewave frequency of 212 Hz was used at 11.12 Vrms. At this frequency the phase shift was 0.03° and the magnitude of the impedance 6.02 Ω. The DC resistance R_e was 5.60 Ω. The ambient temperature during measurement of the impedance and measurement of the step response was 22.5 °C.

Figure 9 shows the long measurement, where the rise in voice coil temperature has been recorded at 1 Hz (1 sample per second) for 5500 seconds. Figure 10 shows the short measurement, measured at a rate of 1 sample every 20 ms for 110 seconds. The first 4096 points of each measurement are then used as target curves for an algorithm to fit the third order thermal model to the measurements. The resolution of the measurement is 0.78 °C.

The result of fitting the model to the measurements is also shown in figures 9 and 10 as the solid curve. The thermal component values for the 170 mm woofer are,

R_1	=	3.927	C_1	=	2.358
R_2	=	1.158	C_2	=	71.49
R_3	=	2.634	C_3	=	664.3

4.3.4 Thermal Components For A 70 mm Drive Unit

This driver is the mid and low frequency unit in the Beolab 6000 loudspeaker from Bang & Olufsen. Two of the units operate in a vented cabinet of 3 litres volume, although only one unit was measured. A 9.00 Vrms sinewave with a frequency 562 Hz was used to measure the step response. The impedance at 562 Hz is 8.43 Ω, -0.38°. The DC resistance of the driver was 7.40 Ω and the ambient temperature during measurement was 23.0 °C. The long measurement is shown in figure 11 and was recorded at 1s intervals. The short measurement (figure 12) was recorded at 5 ms intervals. The measurement resolution was 0.41 °C. In each figure the third order model, with the parameters below, is shown as the solid curve.

R_1	=	5.226	C_1	=	1.073
R_2	=	1.439	C_2	=	21.09
R_3	=	5.005	C_3	=	392.3

4.3.5 Thermal Components For A 19 mm Tweeter

This tweeter is the treble driver in the Beolab 6000 loudspeaker. The unit has a neodymium magnet and fluid in the gap to aid heat dissipation. For measurement of the step response a 2.37 kHz sinewave at 4.11 Vrms was used. The DC resistance of the tweeter is 6.10 Ω and the impedance at the frequency of measurement was 7.70 Ω, 0.60°. The duration of the two measurements where 20 s and 800 s recorded at 5 ms and 200 ms intervals for the short and long measurements respectively. The measurement resolution is 0.50 °C. The data, together with the model, is shown in figures 13 and 14. The thermal component values for the model are given below,

R_1	=	11.98	C_1	=	0.06813
R_2	=	3.065	C_2	=	2.233
R_3	=	47.13	C_3	=	8.259

5. Implementation of the Simulation

As we have seen, it is now possible to model the thermal behaviour of a moving coil loudspeaker and determine the thermal component values in the model. The model can now be used in a simulation. The simulation system developed is described in the next sections.

5.1. Hardware Available to Implement Our Requirements

Hardware available for implementation of the simulation was a digital signal processing system that has previously been used for other projects including Programme Material Analysis [5]. The system consists of a Bang & Olufsen Beosystem 2300 CD player equipped with a digital audio output in S/P DIF format. This signal is input to an audio input interface and format conversion board prior to the digital signal processor (DSP). The DSP comprises of four 40 MHz floating-point Motorola DSP96002 boards from Loughborough Sound Images, UK. The host PC is a Compaq SystemPro LT. The DSP boards are programmed in Motorola assembly code and the host PC in Borland C++.

5.2. Overview of the Simulation

A block diagram of the complete simulation for one audio channel and a single crossover is shown in figure 6. In practice, two audio channels can each be filtered using a four-way crossover system simultaneously, outputting 16 temperatures (2 channels x 4 x voice coil and magnet temperatures) in real time.

5.3. Input Signal & Crossover

The input signal is standard digital audio in S/P DIF format from compact disc at a sampling rate of 44.1 kHz. For the purpose of modelling a crossover in the loudspeaker system, the signal can then be filtered using up to a tenth order filter. This IIR filter is implemented in the form of 5 cascaded biquads, to reduce the effects of coefficient sensitivity, and a gain coefficient. If no crossover is required, then the coefficients can simply be set to zero with a unity gain coefficient. An example of the z-plane transfer function for a fourth order crossover is shown below,

$$H(z) = \left[\frac{1 + a_{01}z^{-1} + a_{02}z^{-2}}{1 + b_{01}z^{-1} + b_{02}z^{-2}}\right]\left[\frac{1 + a_{11}z^{-1} + a_{12}z^{-2}}{1 + b_{11}z^{-1} + b_{12}z^{-2}}\right] gain \qquad \textbf{Eqn 11}$$

5.4. Amplifier Section

For the purposes of modelling an amplifier a simple voltage gain was chosen that scales the voltages to actual level, for example +/- 50 V peak. This is somewhat ideal but it could be argued that amplifiers used are sufficiently linear and of low distortion for this simple model to be accurate.

However, high temperatures within a loudspeaker are generated by large voltages being applied and this often implies clipped output signals from the amplifier. For this reason, the simple amplifier model above has been extended by including a hard clipping function, that clips the signal at a specified level, for both positive and negative signal.

The signal now represents the voltage as a function of time, v(t), appearing at the terminals of the loudspeaker.

5.5. Calculation Of Signal Power - An Admittance Filter

To calculate the power applied to the loudspeaker and hence the thermal model, one needs a knowledge of the instantaneous current flowing in the voice coil. If the loudspeaker presented a purely resistive load that was independent of temperature then the power could be calculated from the squared voltage divided by this resistance. However, the loudspeaker has a complex impedance that is dependant upon frequency (f) and temperature (T). Figure 15 shows a typical impedance curve magnitude, Z(f,T).

The current flowing in the voice coil can be calculated by filtering the voltage with the inverse of the impedance, or the admittance function Y(f,T). The admittance curve magnitude for the impedance in figure 15 is shown in figure 16. The current in the voice coil is now given by,

$$i(t,f,T) = v(t)\frac{1}{Z(f,T)} = v(t)Y(f,T) \qquad \textbf{Eqn 12}$$

Where the current in the voice coil is a function of time, frequency and temperature. It can be seen that the admittance function is dependant upon temperature. However, data given by Gottfried [2], shows that the change due to temperature is almost entirely a shift in DC resistance R_e. Therefore, the simulation considers a single impedance measurement made at normal ambient temperature and introduces the temperature dependence through the change in R_e. This method then avoids having to measure the loudspeaker's impedance at many temperatures.

The power flowing into the thermal model in then given by,

$$P(t,f,T) = [i(t,f,T)]^2 R_e(T) \qquad \textbf{Eqn 13}$$

Where, T is initially the start temperature or ambient and then subsequently the voice coil temperature as calculated by the model. R_e at a temperature T for a voice coil wound from copper wire is given by,

$$R_e(T) = R_e(T_0)[1 + \alpha(T - T_0)] \qquad \textbf{Eqn 14}$$

α is the temperature coefficient of copper (0.004 K^{-1}) and $R_e(T_0)$ is the DC resistance of the coil at ambient temperature (or the temperature at which the impedance measurement was made).

5.6. Implementing the Admittance Filter

Filtering the voltage signal v(t) to obtain the current is performed in the simulation with a sixth order IIR filter. This is sufficient to model the admittance curve for a vented box loudspeaker. Initially a sixth order transfer function in the form of three cascaded biquads is fitted to the impedance measurement. This z-plane transfer function may be written as,

$$Z_1(z) = \left[\frac{1 + a_{01}z^{-1} + a_{02}z^{-2}}{1 + b_{01}z^{-1} + b_{02}z^{-2}}\right]\left[\frac{1 + a_{11}z^{-1} + a_{12}z^{-2}}{1 + b_{11}z^{-1} + b_{12}z^{-2}}\right]\left[\frac{1 + a_{21}z^{-1} + a_{22}z^{-2}}{1 + b_{21}z^{-1} + b_{22}z^{-2}}\right] gain_1 \qquad \textbf{Eqn 15}$$

Which can be written as,

$$Z_1(z) = \left[\frac{a}{b}\right] gain_1 \qquad \textbf{Eqn 16}$$

With the increased DC resistance, the new impedance will be,

$$Z_2(z) = Z_1(z) + \delta R_e = \left[\frac{a}{b}\right] gain_1 + \delta R_e = \frac{a(gain_1) + b(\delta R_e)}{b} \qquad \textbf{Eqn 17}$$

Then the new admittance transfer function is given by,

$$Y(z) = \frac{1}{Z_2(z)} = \frac{b}{a(gain_1) + b(\delta R_e)} = \left[\frac{b}{c}\right] gain_2 \qquad \textbf{Eqn 18}$$

To implement the admittance filter at the new value of R_e, it is necessary to reduce equation 18 into the form or three cascaded biquads to avoid sensitivity to coefficient round-off error. However, c is a sixth order polynomial which must be factored into its roots in order to find the three quadratics to form the biquads. With a knowledge that the roots must be real or in complex conjugate pairs, the roots are found within the simulation using an efficient iterative process based on a method by Laguerre [6].

In practice, prior to running a simulation, a data set is created that contains coefficients for 121 admittance filters in steps of 2 °C from a start temperature T_s to T_s+240 °C.

5.7. Down-sampling

The processing and calculations in the simulation so far are implemented at a sample rate of 44.1 kHz. This sample rate is much too high for accurate implementation of the thermal model as described next. To avoid this problem the power signal is down-sampled to a frequency of 44100/16384 or 2.692 Hz. The down-sampling is performed using a summation of the power for the previous 16384 samples.

5.8. Implementation of Third Order Thermal Model In The Simulation

The temperatures given by the thermal model are calculated from the input power signal by equations 5, 6 and 7. It is seen that these are simply third order s-plane transfer functions defining low-pass filters. For the simulation, these have been implemented as digital IIR filters composed of two cascaded biquads for efficient implementation. The coefficients for the filters are calculated from their s-plane equivalents using the bilinear transform method.

Due to the long time constants of the thermal system, the cut-off frequencies for the filters are extremely low relative to a sampling frequency of 44.1 kHz giving rise to filter coefficients that are very close to the unit circle and thus the filter is sensitive to coefficient round-off error. Therefore, to implement the filters accurately, the sampling frequency has been reduced by down-sampling (section 5.7). The new sampling rate gives a Nyquist frequency of ~1.35 Hz and the effect of this when implementing the transfer function of the tweeter voice coil is shown in figure 17. This is the worst case situation as the time constant of the voice coil is the shortest. The desired transfer function is shown as the dotted curve and its digital equivalent as the solid curve. To evaluate the error introduced by implementing the digital filter with the new sampling frequency, consider applying a power signal as shown in figure 18 to the tweeter. We see that this represents a step function modulated with a sinewave of the same amplitude as the step and at the Nyquist frequency. The average power is indicated by the dotted curve and is a step function. When steady state conditions prevail, let the average power have heated the voice coil to a real life maximum of 300 °C. The desired filter response in figure 17 then indicates that the sinewave will cause a ripple in the steady state temperature of 0.3 °C peak-peak (300 °C - 30 dB) whereas the digital implementation will reject this ripple but will give the correct average temperature of 300 °C. Hence, the maximum error is 0.15 °C in 300 °C or 0.05%.

With the new sampling frequency, the simulation will calculate voice coil and magnet temperatures approximately three times each second (44100/16384 per second). The temperature T_g could easily be calculated, although it has not been in this simulation system.

6. Running The Simulation

Through a measurement of the step response of the loudspeaker drive units it is possible to determine the thermal component values of the third order thermal model for each unit. The results for three units have been presented. In order to make any simulations it is necessary to compile a data set for a particular simulation. This data set contains,

- Filter coefficients for up to a tenth order crossover filter.
- A peak amplifier output voltage.
- A clipping voltage (to implement hard clipping of the signal if necessary).
- The ambient temperature at which the impedance and step responses were measured.
- The DC resistance of the voice coil at the above ambient temperature.
- An ambient temperature at which to start the simulation.
- 121 sets of filter coefficients for the admittance filter generated from the coefficients for a sixth order filter modelling the impedance curve of the drive unit and the temperature coefficient of the voice coil material.
- Filter coefficients for the fourth order filters to implement the thermal model.

The data set is created using software written for the simulation system in Borland C++ and takes a matter of seconds to be compiled. The data can now be used for simulation of the temperatures of the particular loudspeaker at any time and for any programme material. If a new crossover is required for example, a new data set can simply be made.

7. Simulations And Measurements With Music

For verification of the simulation system, the temperature of the voice coil and magnet for each of the three drive units mentioned has been measured and simulated. The Pink Floyd album "Dark Side of the Moon" on EMI Record Ltd was used. The period for the measurements and simulations was 3000 s. This period includes the complete album and several minutes extra. The temperatures were recorded at 1 s intervals for the measurements. The left audio channel was measured. For each drive unit the complete 3000 s period is presented together with a 600 s period from within the complete period to show detail.

Figures 19 - 22 show results for the 170 mm woofer. The measured voice coil temperature, obtained using the thermocouple wound into the voice coil, is shown in figures 19 and 21. No crossover filter was used and the peak, unclipped voltage from the amplifier was 45.7 V. Figures 20 and 22 show the simulated temperature of the voice coil and magnet.

Figures 23 - 26 show results for the 70 mm driver. The music signal in this case was low pass filtered at 3 kHz with a 24 dB/octave Linkwith-Riley filter. No high pass filter was used. The consequence of not using a high pass filter is that bleed-through of the music signal appears on the measurement. This is seen as noise on the measurements in figures 23 and 25. The unclipped peak voltage on the driver was 38.6 V. The simulation of the temperatures is shown in figures 24 and 26.

Measurements and simulations for the 19 mm tweeter are displayed in figures 27 to 30. The signal to the tweeter was high pass filtered at 3 kHz with a 24 dB/octave Linkwith-Riley filter. The unclipped peak voltage on the tweeter was 38.6 V.

8. Discussion Of Results

Firstly, let us consider the difference in resolution between the simulations and the measurements for verification of the simulations. There are two differences, in time and in temperature:

- **Resolution In Time:**

The music signal measurements of voice coil temperature are written to file at intervals of 1 s. Each of these temperatures is an average of 10000 samples made by the datalogger each second. Therefore, the measurements are short term averages. The simulations however, calculate temperatures at intervals of 0.372 s. The effect of this time discretisation will cause differences in the short term peak and short term low temperatures. The simulation will indicate slightly higher peak temperatures and slightly lower low temperatures due to the measurement to some extent averaging away these short term peaks and dips.

- **Resolution In Temperature:**

The simulated temperatures written to file by the simulation system and are rounded to the nearest 0.01 °C. The temperature resolution of the measurements is limited by the 8 bit datalogger. In the case of the 170 mm woofer, this resolution is 0.78 °C and for the 70 mm unit and tweeter is 0.41 °C and 0.50 °C respectively. This quantisation is seen as 'steps' in the measurements, particularly visible for the 600 s periods of the music signal and for the tweeter measurements.

The results for each of the three loudspeaker units are now discussed.

- **Results for the 170 mm Woofer:**

Considering the 3000 s period in figures 19 and 20, it is seen that the simulation calculates temperatures above approximately 35 °C to within 1.5 °C of the measurement. Below 35 °C, when the voice coil cools during periods of relative quiet in the music, the simulation indicates lower temperatures than the measurement but the error is never greater than 2 °C. This is caused by the voice coil in the measurement cooling to its surrounding temperature, the temperature of the magnet close to the voice coil. However, the simulation assumes the system is composed of lumped elements and thus the voice coil will cool to a temperature closer to the average temperature of the magnet system. Figures 21 and 22 show the 600 s period and indicate that the simulation reveals more details in the temperature than it has been possible to measure.

- **Results for the 70 mm Unit:**

The results are shown in figures 23 to 26. The measurement circuit used to extract the DC resistance of the voice coil and hence the temperature is essentially a first order low pass filter will a very low cut off frequency. The loudspeaker was driven with the music signal low pass filtered below 3 kHz. Consequently, low frequency music signals are passed through the measuring circuit resulting in noise on the measurement. Hence, it is not possible to compare detail although one sees that the simulated temperature lies within the noisy measurement.

At the end of the 3000 s period when the music signal has ended (when there is no noise on the measurement) the simulation is 0.5 °C below the measurement.

• **Results for the 19 mm Tweeter:**
Figures 27 to 30 show the simulation of the tweeter temperatures to be very close to the measurement. However, the resolution in the measurement causes detail to be lost, the result being that the simulation indicates higher short term peaks in the voice coil temperature. Comparing figures 29 and 30, the 600 s period, we see the simulated temperature showing every detail of the temperature fluctuations, within 0.7 °C of the measurement. During periods of quiet in the music signal it is seen that the voice coil temperature cools to the magnet temperature in the simulation. This also closely matches the cooling of the measured voice coil temperature. The simulation is much more accurate during periods of quiet for the tweeter because the tweeter thermal behaviour is much closer to a lumped system. Also, the tweeter has fluid in the gap giving more efficient heat dissipation from voice coil to magnet.

Comparing the complete 3000 s period for the tweeter, figures 27 and 28, the simulation is very close to the measurement for the initial 1600 s. After this time the simulation slowly falls below the measurement to 1.5 °C lower at the end of the period. This is most likely caused by a small increase in the ambient temperature towards the end of the measurement period.

8.1. Possible Errors in the Simulation Process

• **Possible errors in the measurement of the step response:**
• During measurement of the step response for derivation of the thermal component values, a sinewave is applied at a frequency of zero phase shift. However, the phase shift may not be exactly zero and also during the long measurements, changes in the loudspeaker parameters may cause the phase to change. This will cause small variations in the power applied.
• The measuring circuit used to measure changes in the DC resistance of the voice coil has itself a step response. However, the time constant of the measuring circuit is approximately 0.1 s and hence is still small relative to the time constant of the tweeter voice coil.
• The ambient temperature during measurements may change and small variations may be unnoticed.
• Some power during the measurement will be dissipated in the mechanical resistance R_m, however this is somewhat taken into account (section 4.3).

• **Possible errors in the simulation:**
• The amplifier model in the simulation assumes no distortion of the signal unless hard clipping is implemented.
• The admittance filter used for calculation of the current flowing in the voice coil is a sixth order digital filter calculated from the measured impedance curve of the loudspeaker. The peaks in the impedance and the high frequency increase caused by L_e are modelled using biquad sections. Hence, there will be differences between the digital filter and the true impedance.
• The simulation assumes the increase in temperature causes a simple increase in DC resistance R_e.
• During operation of the loudspeaker there is forced air cooling of parts of the system by the movement of the cone. This effect is not included directly in the simulation, however, it is included during measurement of the step response of the loudspeaker and derivation of the thermal component values.

• The thermal model is implemented as digital filters and errors caused by this are evaluated in section 5.8.
• The thermal model assumes the thermal system is composed of lumped elements. Errors caused by this will be most obvious for drive units with large magnets. This has been somewhat included by using a third time constant close to that of the voice coil.

9. Conclusion

From the results for the music signals it is seen that the simulations are accurate, with errors between the measurements and simulations being less than 2 °C. For the 19 mm tweeter, which has given the most accurate results, the error on average being approximately 0.7 °C. It is also possible to conclude that the simulation reveals more detail in the temperature of the voice coil than it has been possible to measure.

It can also be concluded from the results that errors introduced by non-linearities, forced cooling, and the effects of the cabinet are small. This is because the simulation is based on a measurement that includes, to a large extent, these effects. Largely, any errors in the system come from errors in the measurement of the step response of the loudspeaker and calculation of the thermal component values. Calculational errors in the simulation are very small.

With an accurate means of simulating the temperatures in a loudspeaker, it is now possible to use this knowledge to eliminate thermal compression and to protect the drive units from damage caused by the heating effect of the signal. Also, with a simulation that does not require connection to the loudspeaker, any investigations into power handling or the development of protection systems can be made without damaging any drive units.

10. Acknowledgements

Many thanks for their assistance to Gert Munch and Jan Abildgaard Pedersen of Bang & Olufsen and Knud Bank Christensen, originally of Bang & Olufsen but now of TC Electronic, Aarhus Denmark.

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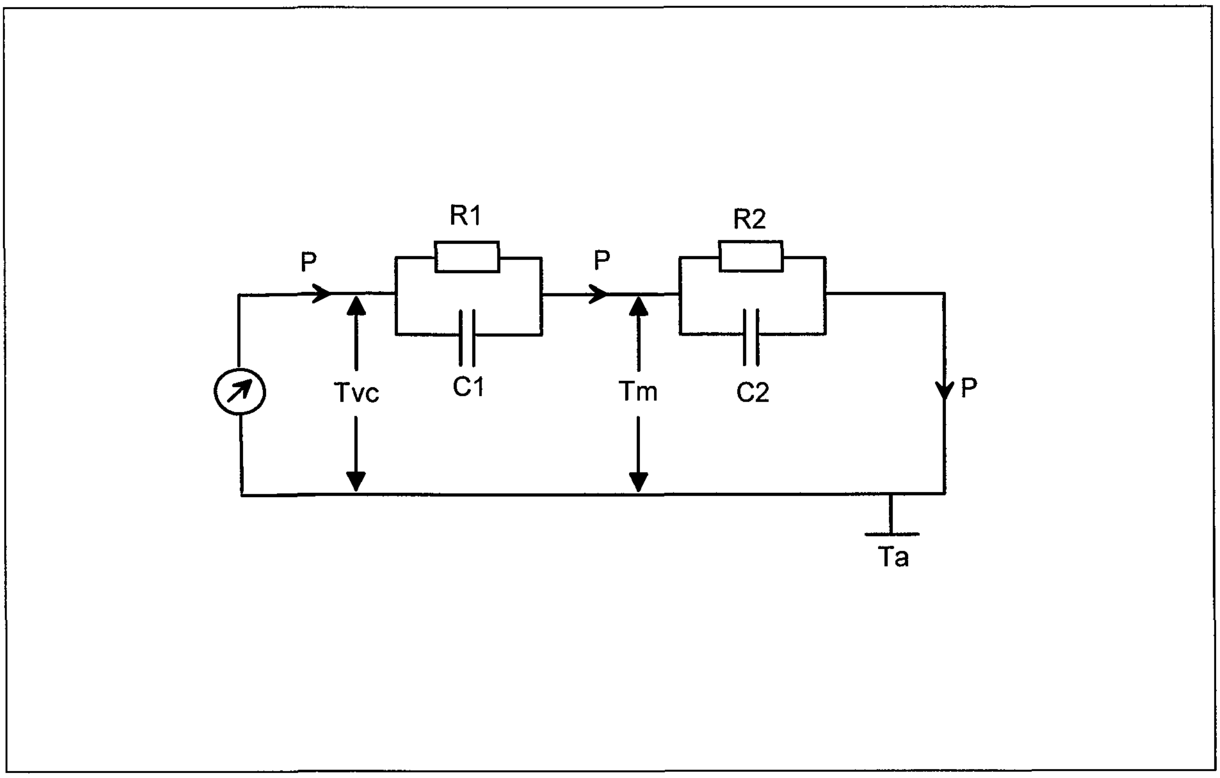

Figure 1 Previously used second order thermal model of a moving coil loudspeaker.

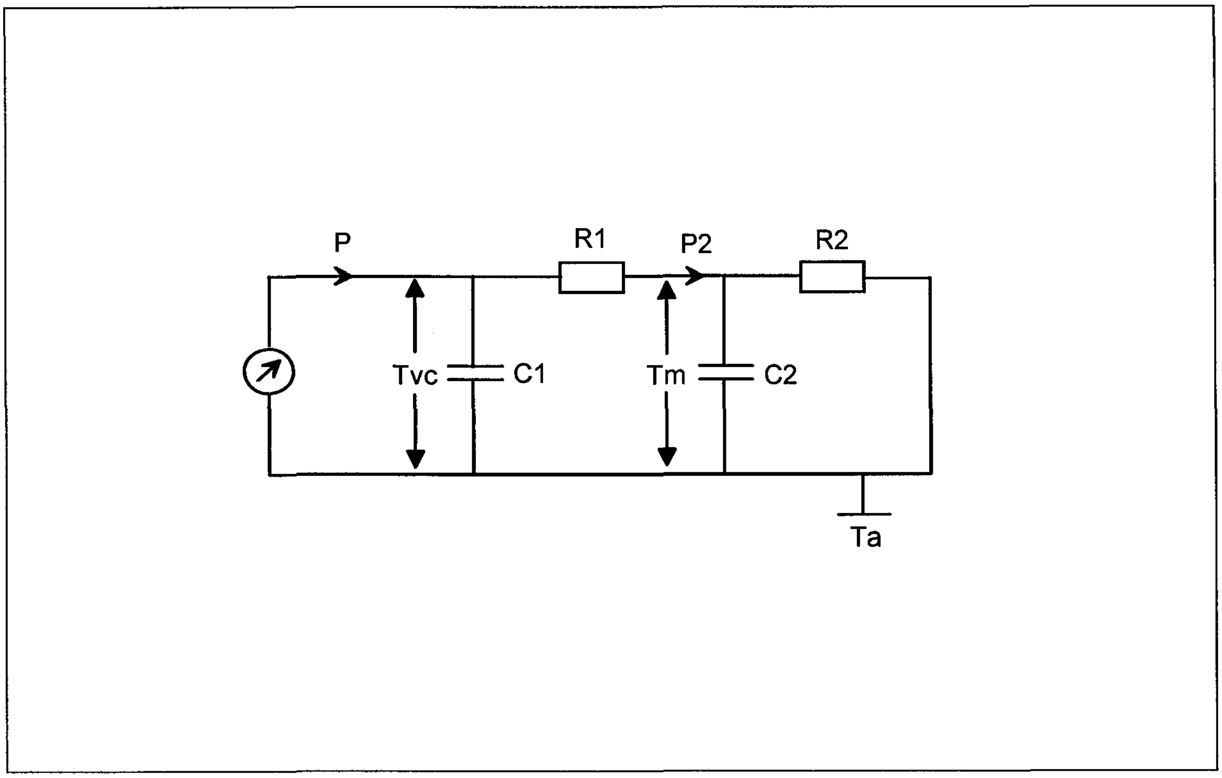

Figure 2 New second order thermal model of a moving coil loudspeaker.

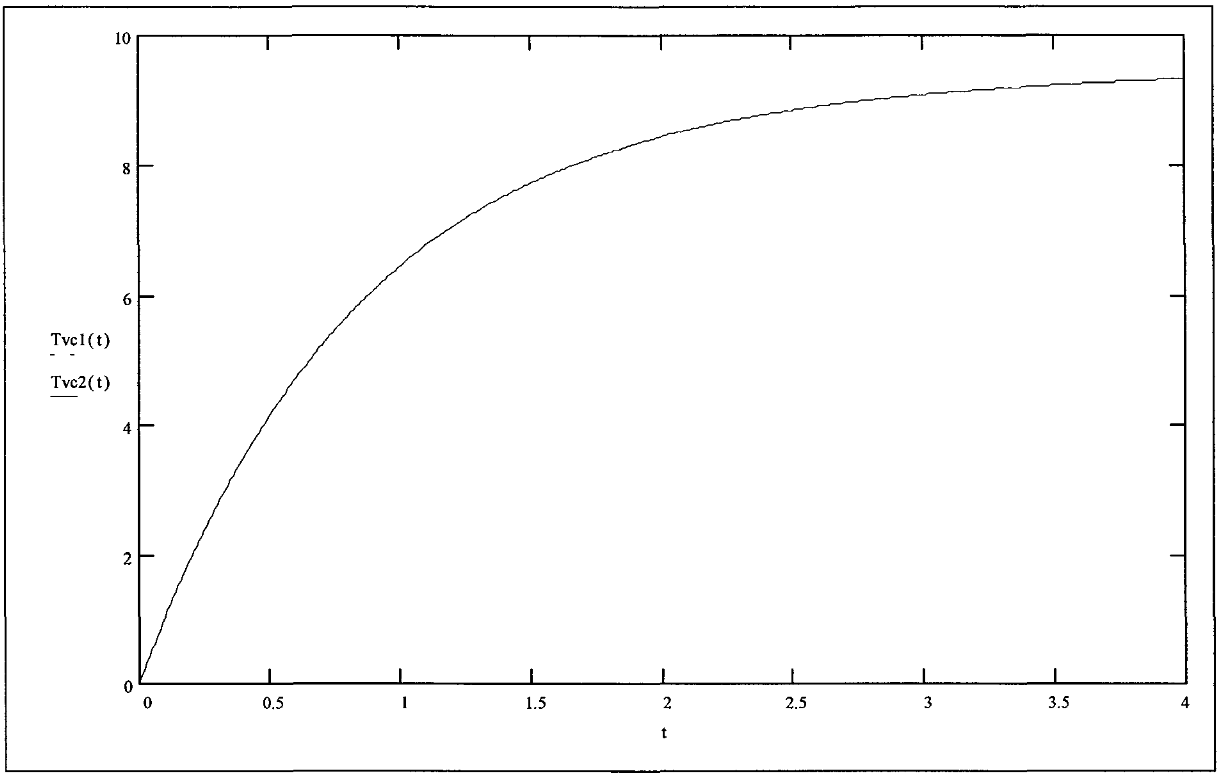

Figure 3 Voice coil step responses for each second order thermal model (19 mm tweeter).

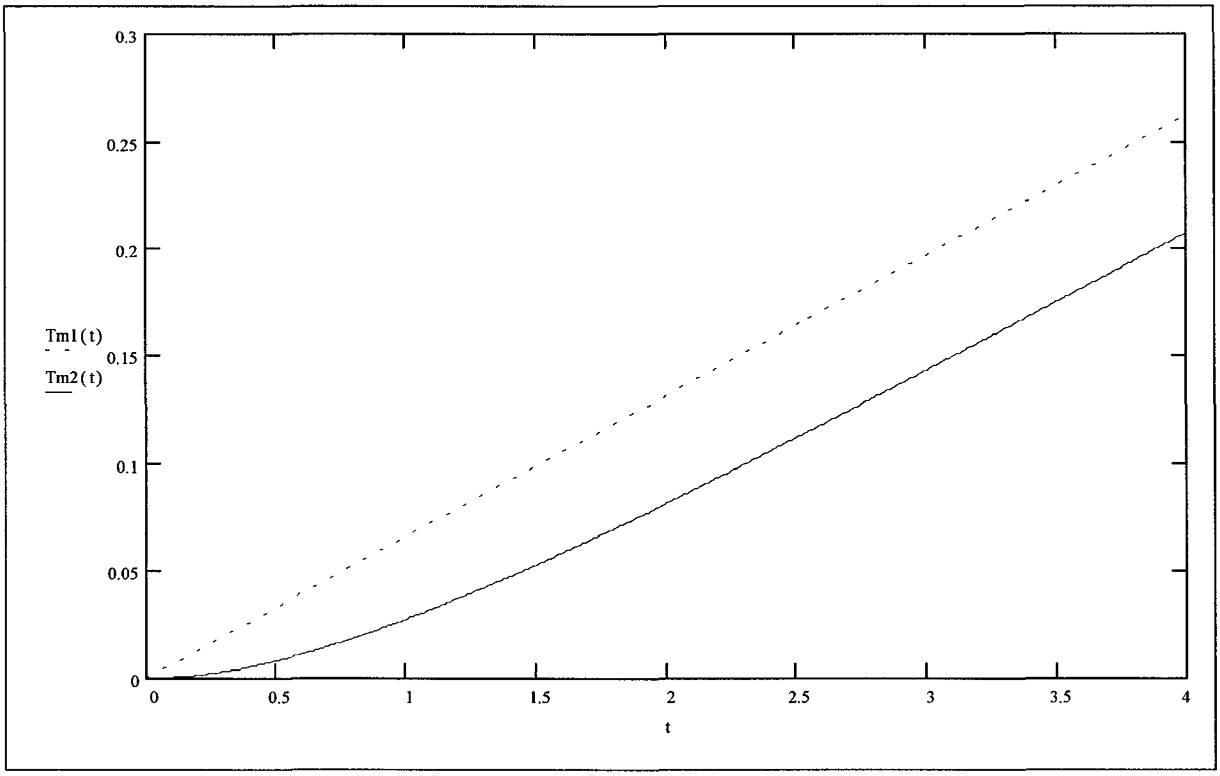

Figure 4 Magnet step responses for each second order thermal model (19 mm tweeter).

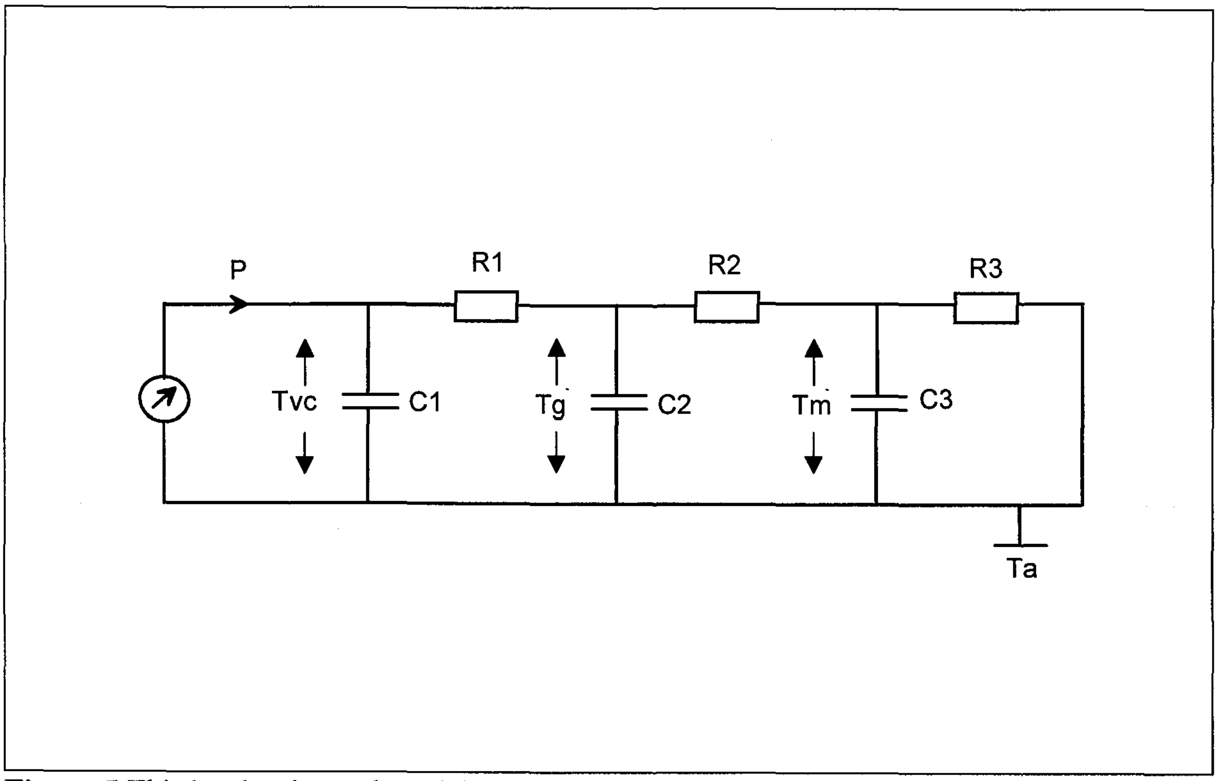

Figure 5 Third order thermal model.

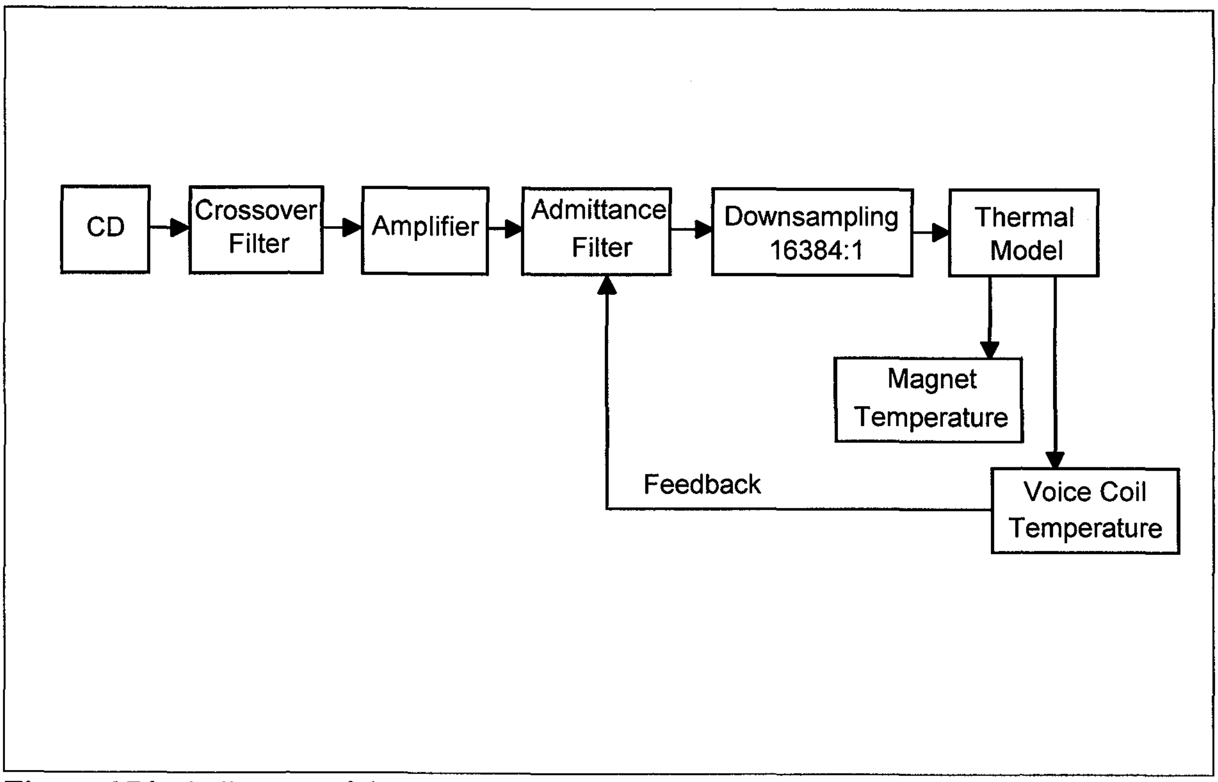

Figure 6 Block diagram of the simulation system.

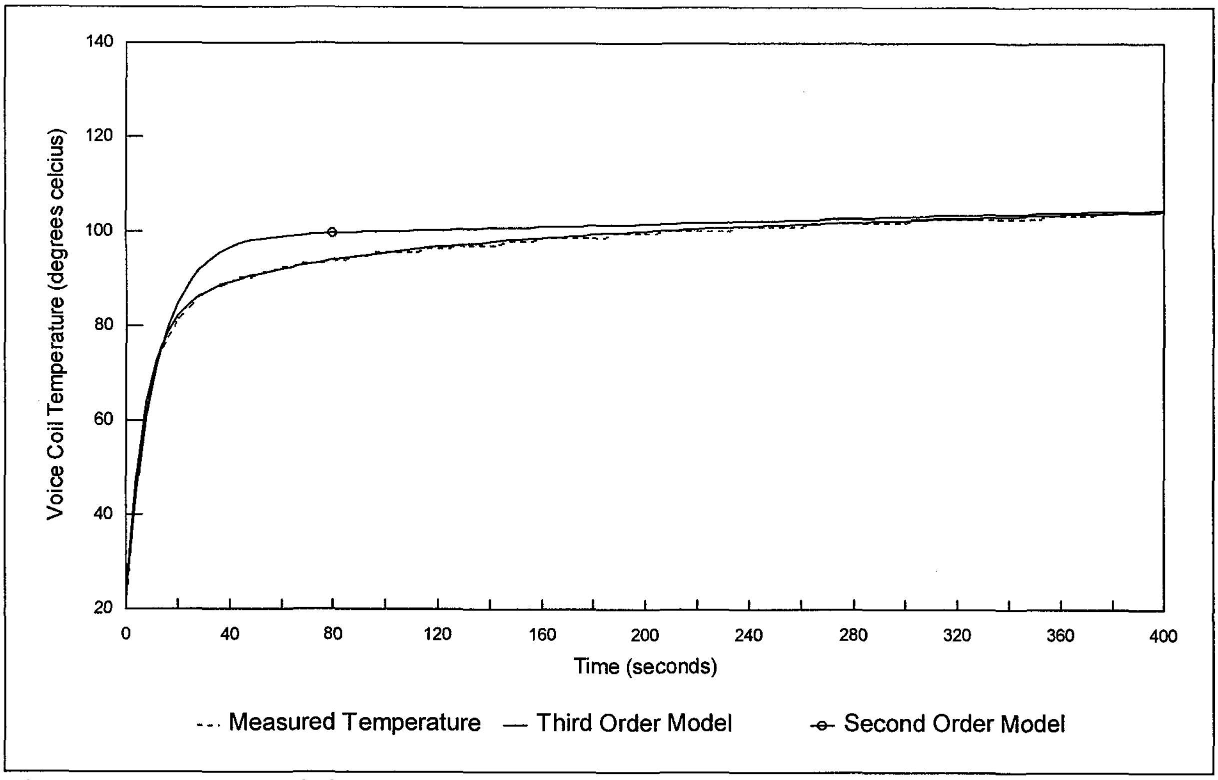

Figure 7 Measured and simulated step responses for 170 mm woofer.

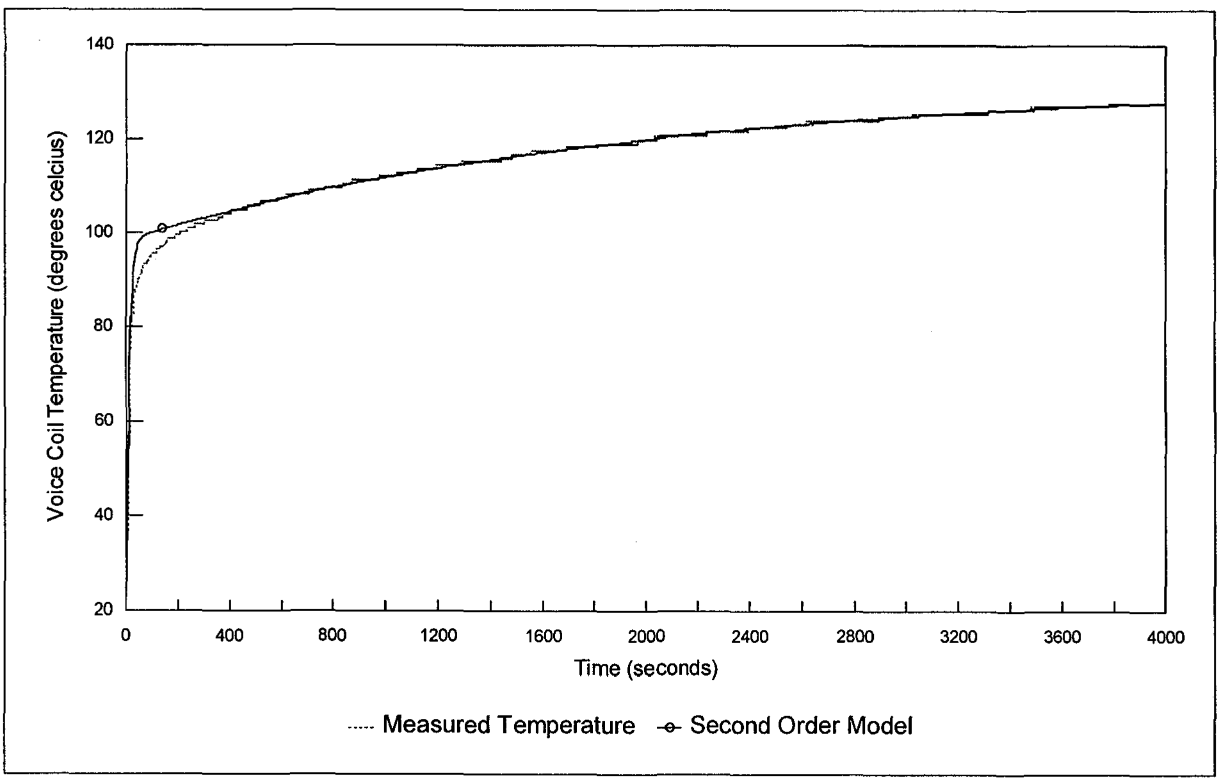

Figure 8 Measured step response and second order model (170 mm woofer).

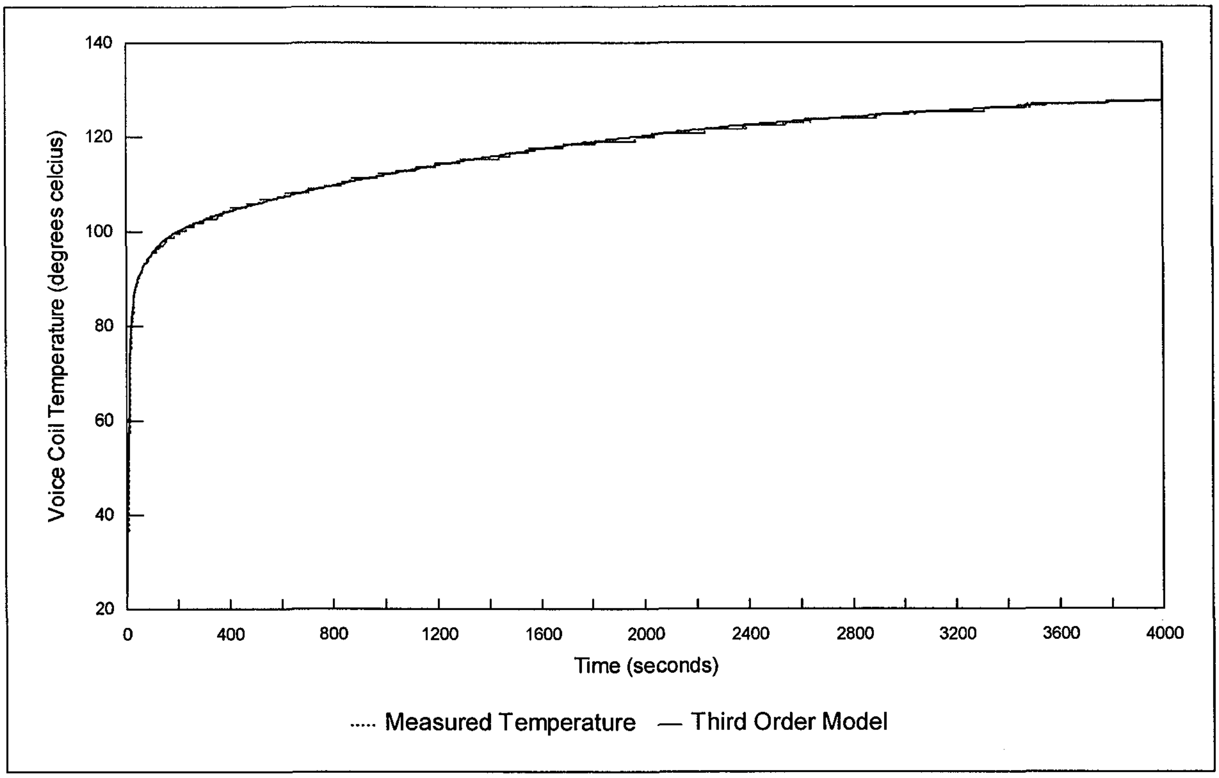

Figure 9 Measured step response and third order model (long measurement, 170 mm woofer).

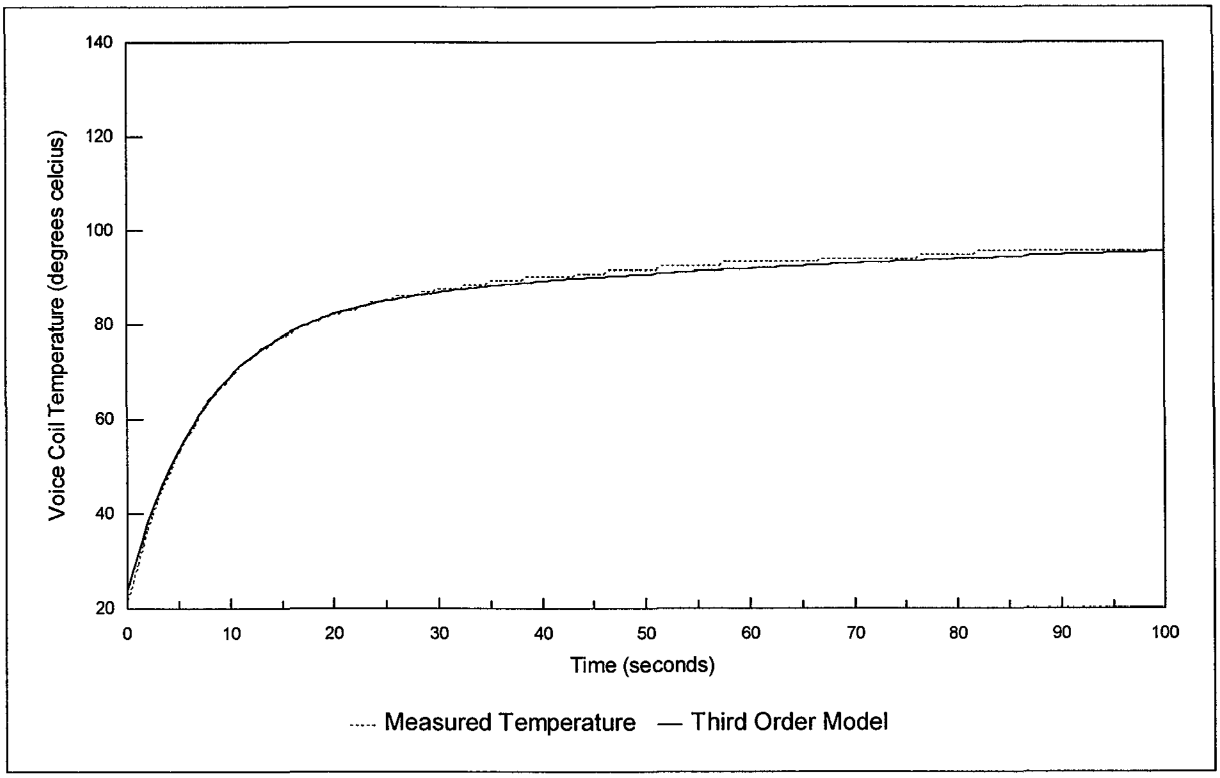

Figure 10 Measured step response and third order model (short measurement, 170 mm woofer).

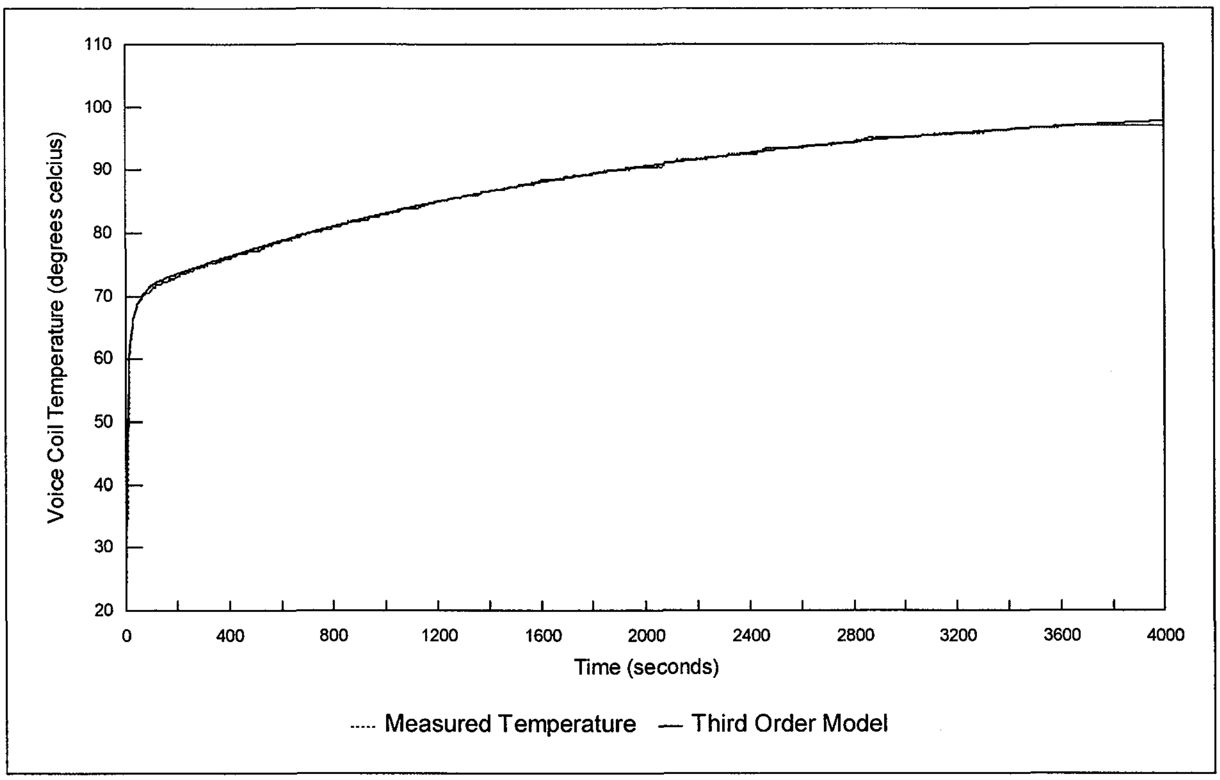

Figure 11 Measured step response and third order model (long measurement, 70 mm unit).

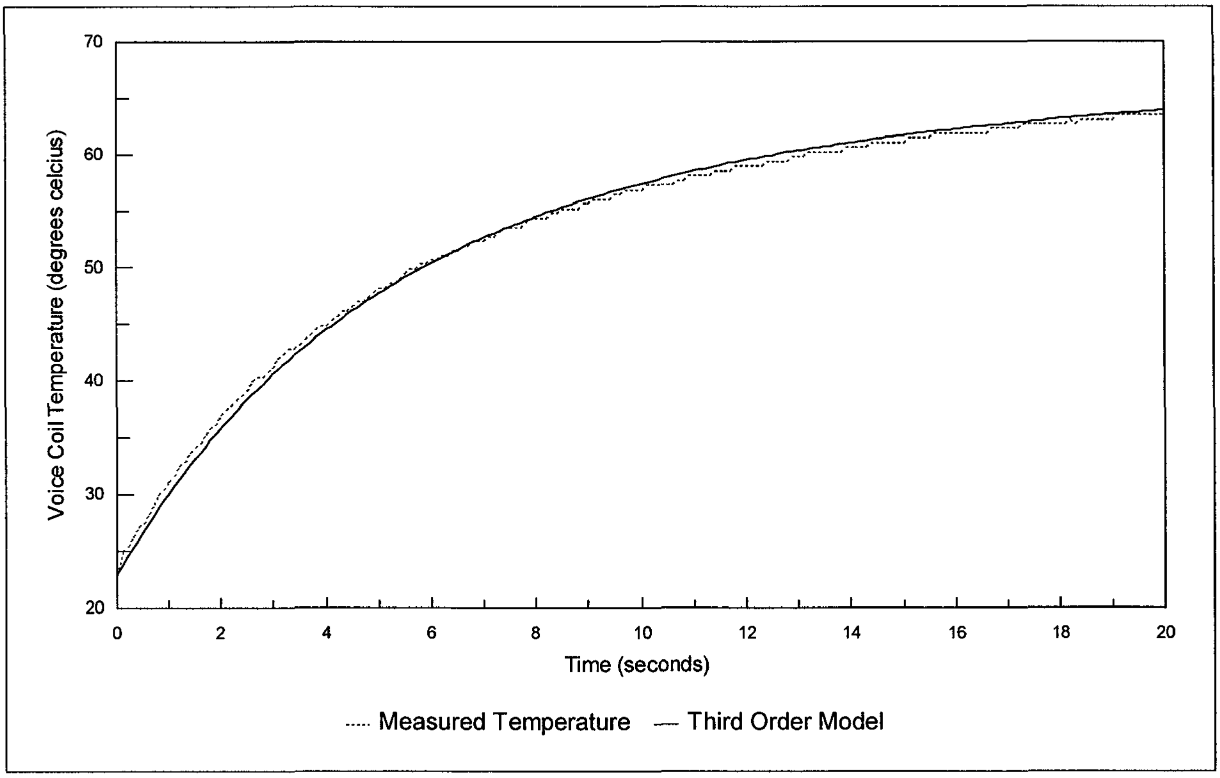

Figure 12 Measured step response and third order model (short measurement, 70 mm unit).

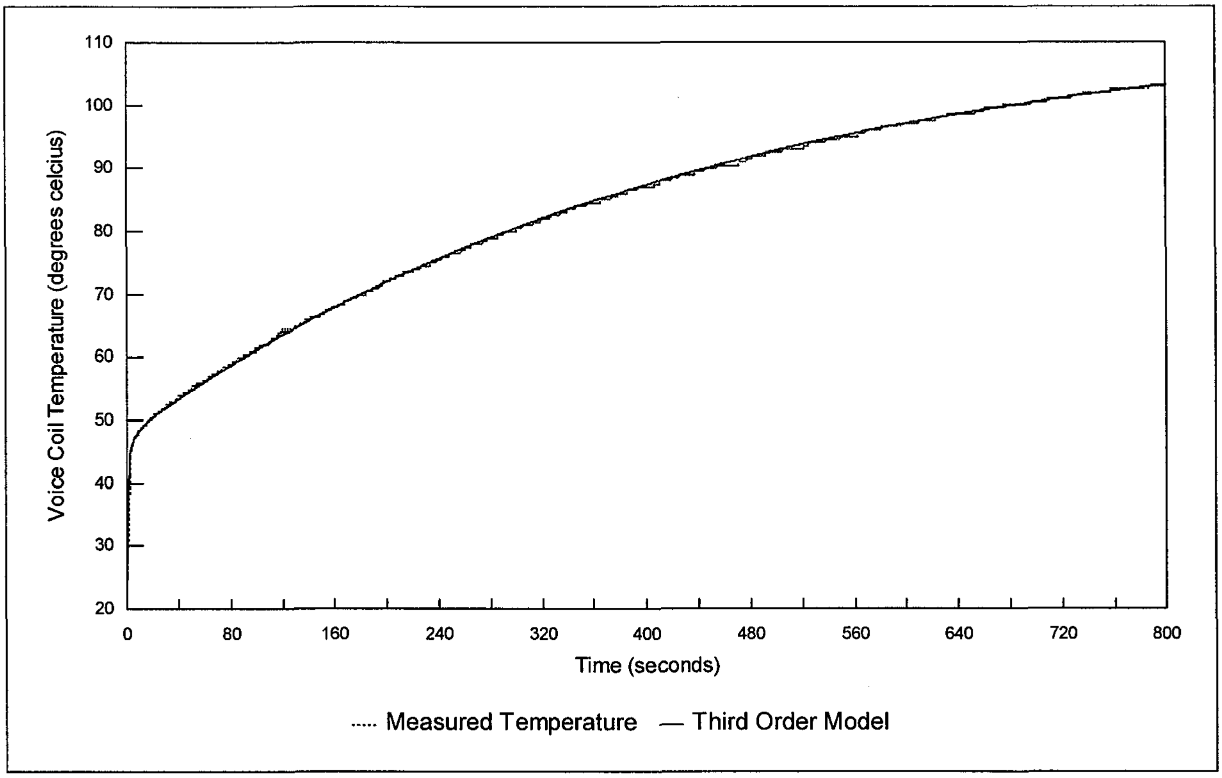

Figure 13 Measured step response and third order model (long measurement, 19 mm tweeter).

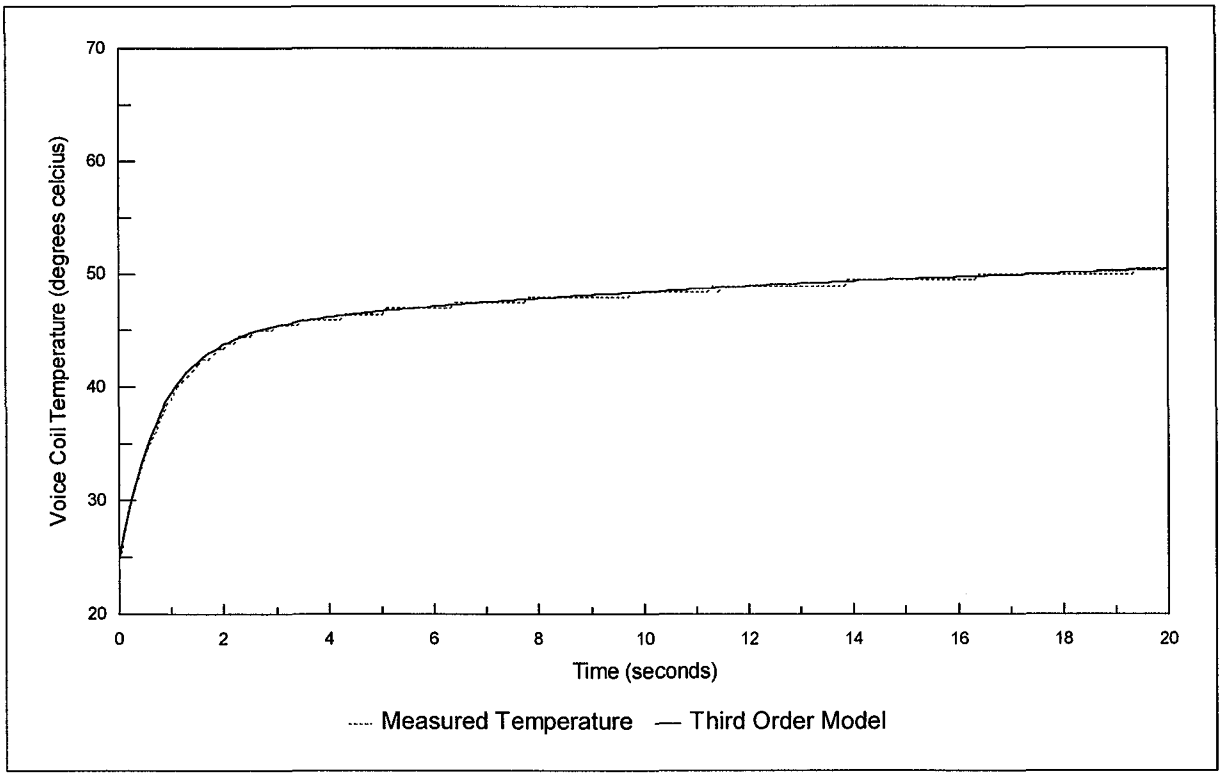

Figure 14 Measured step response and third order model (short measurement, 19 mm tweeter).

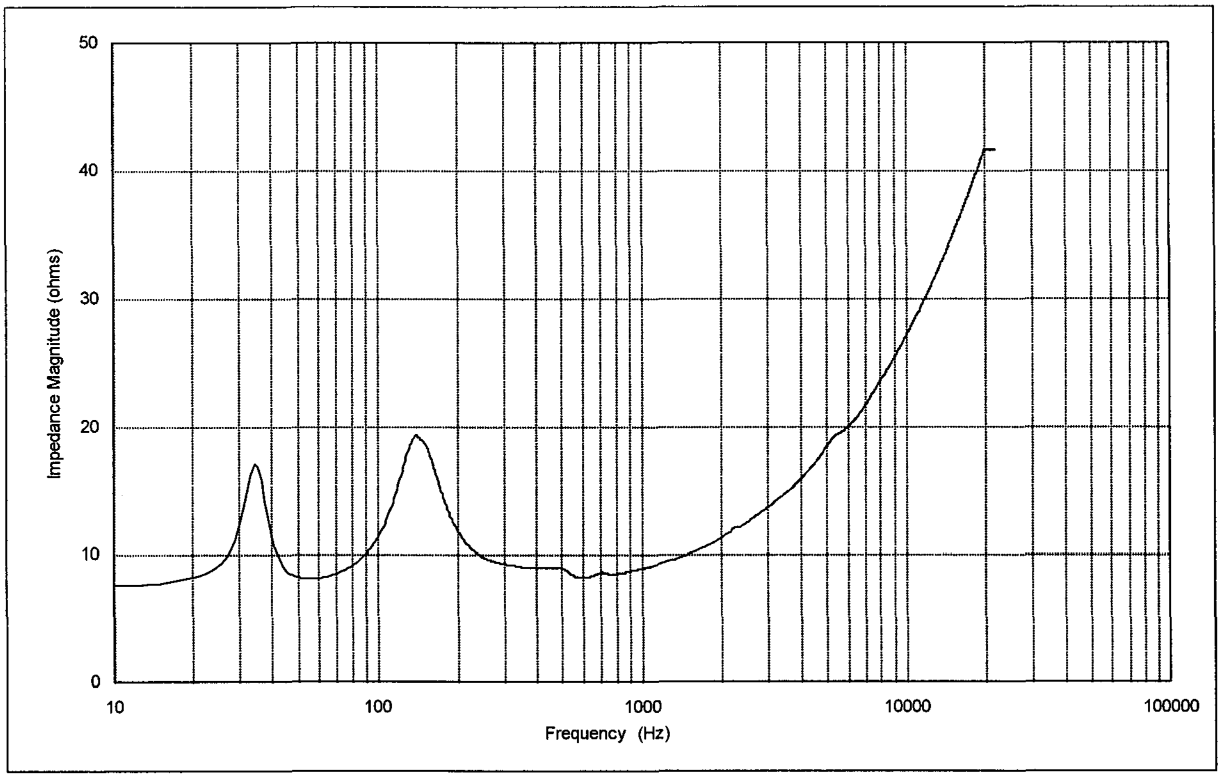

Figure 15 Typical impedance of a loudspeaker unit in a vented cabinet.

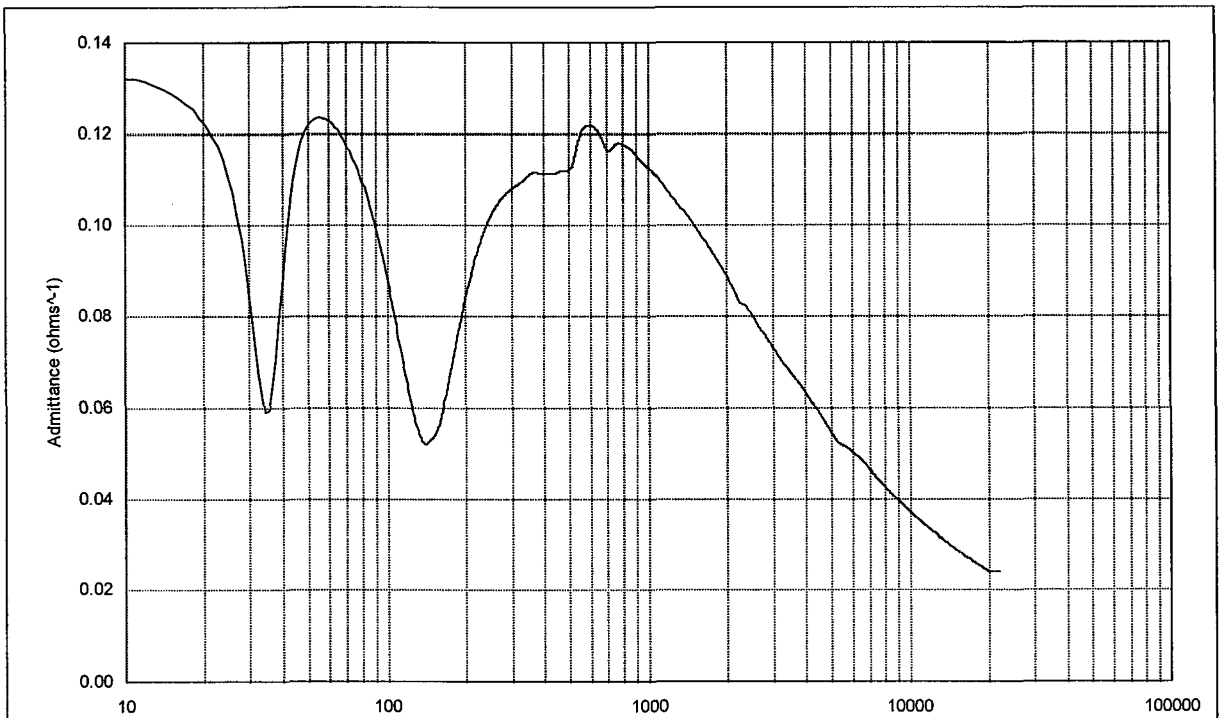

Figure 16 Admittance function.

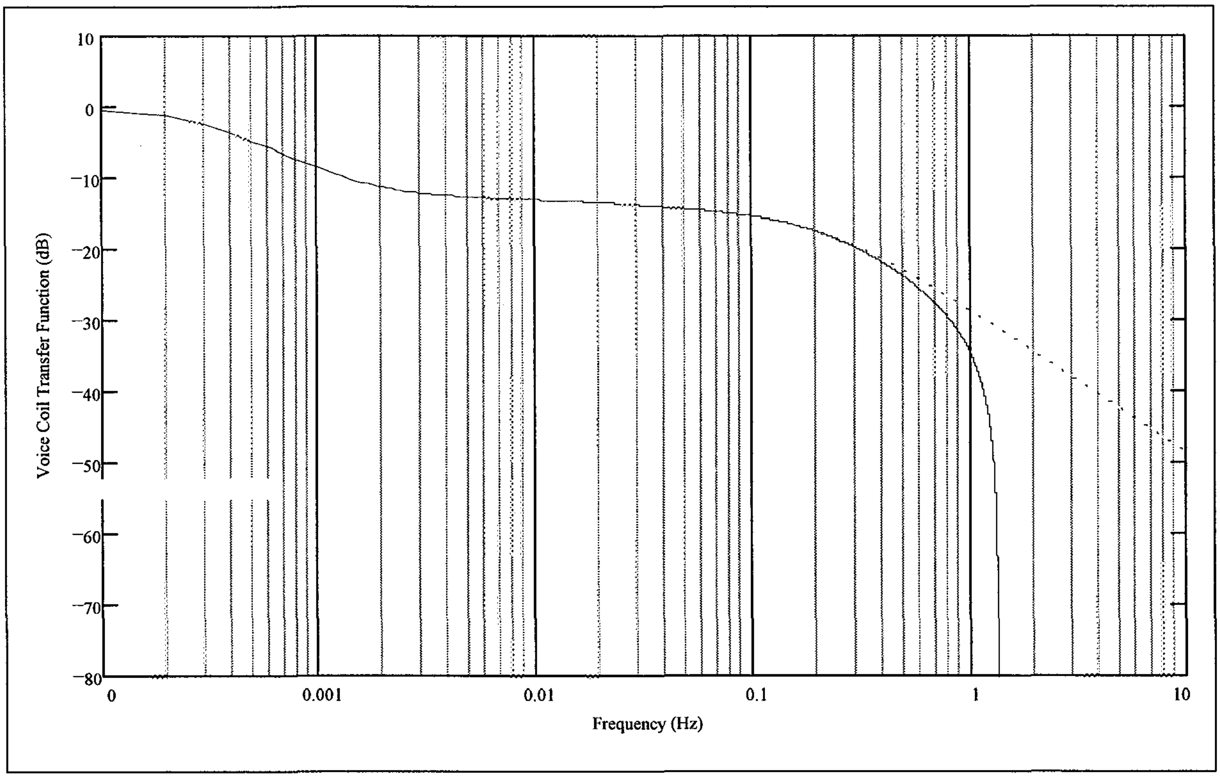

Figure 17 Transfer function of thermal model for the 19 mm tweeter voice coil.

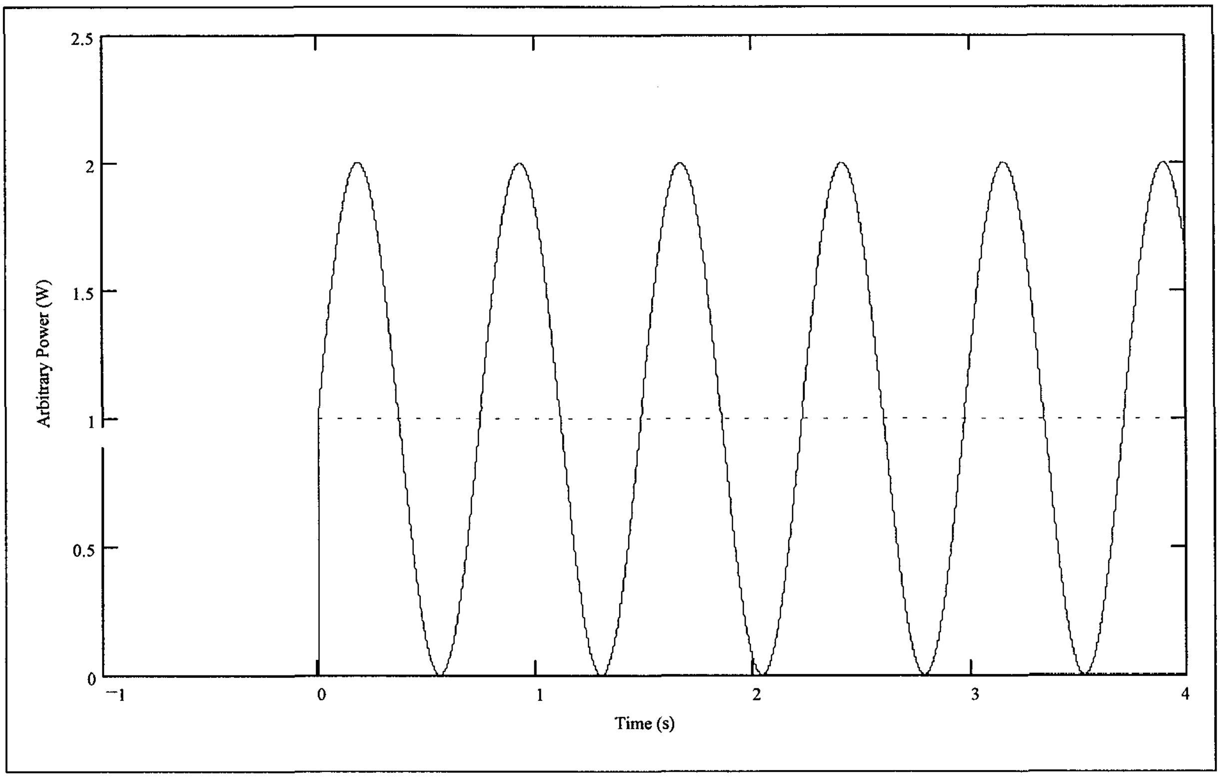

Figure 18 Example of an applied power signal to indicate error caused by digital filter implementation of thermal model for tweeter voice coil.

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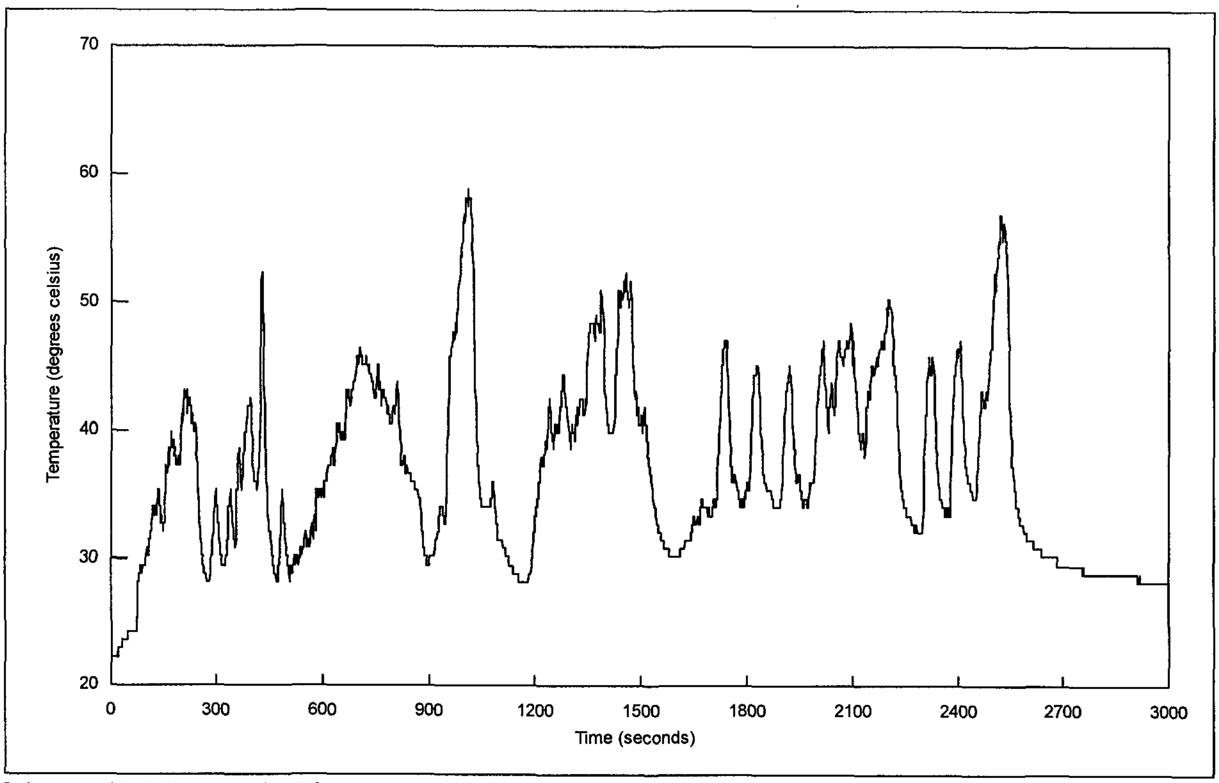

Figure 19 Measured 170 mm woofer voice coil temperature (3000 s period).

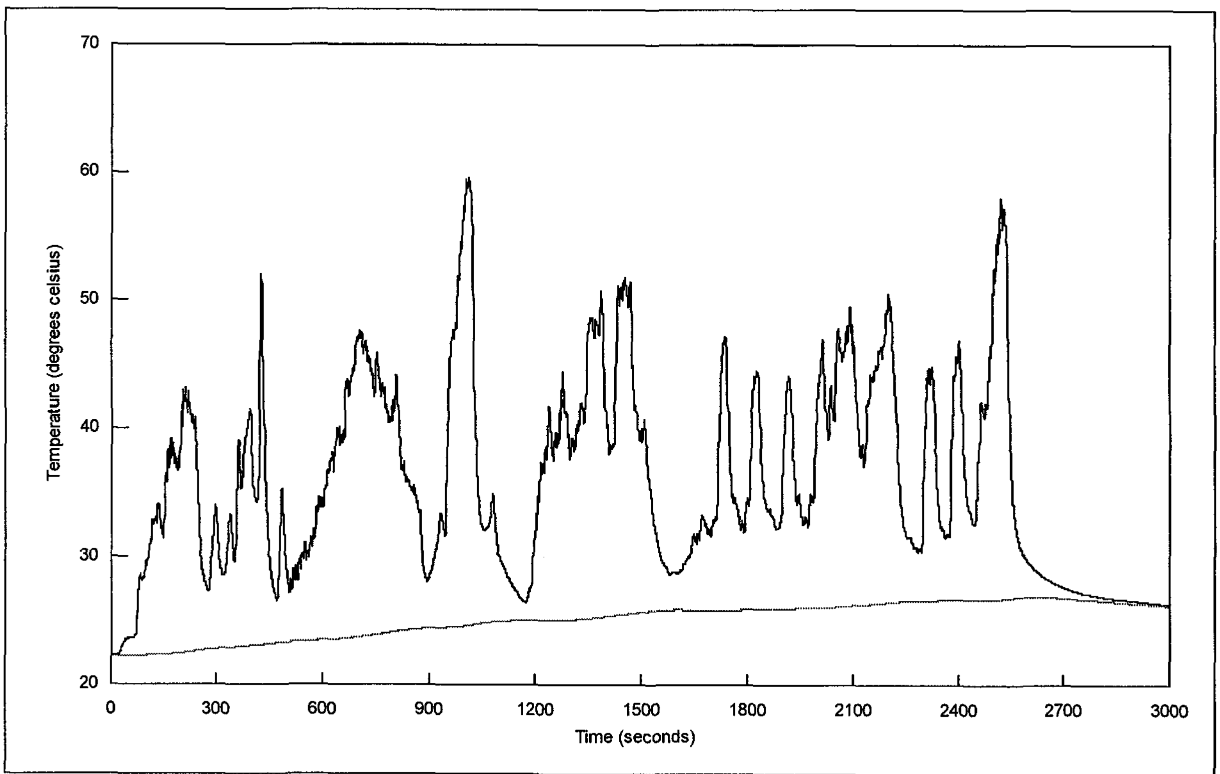

Figure 20 Simulated 170 mm woofer voice coil and magnet temperatures (3000 s period).

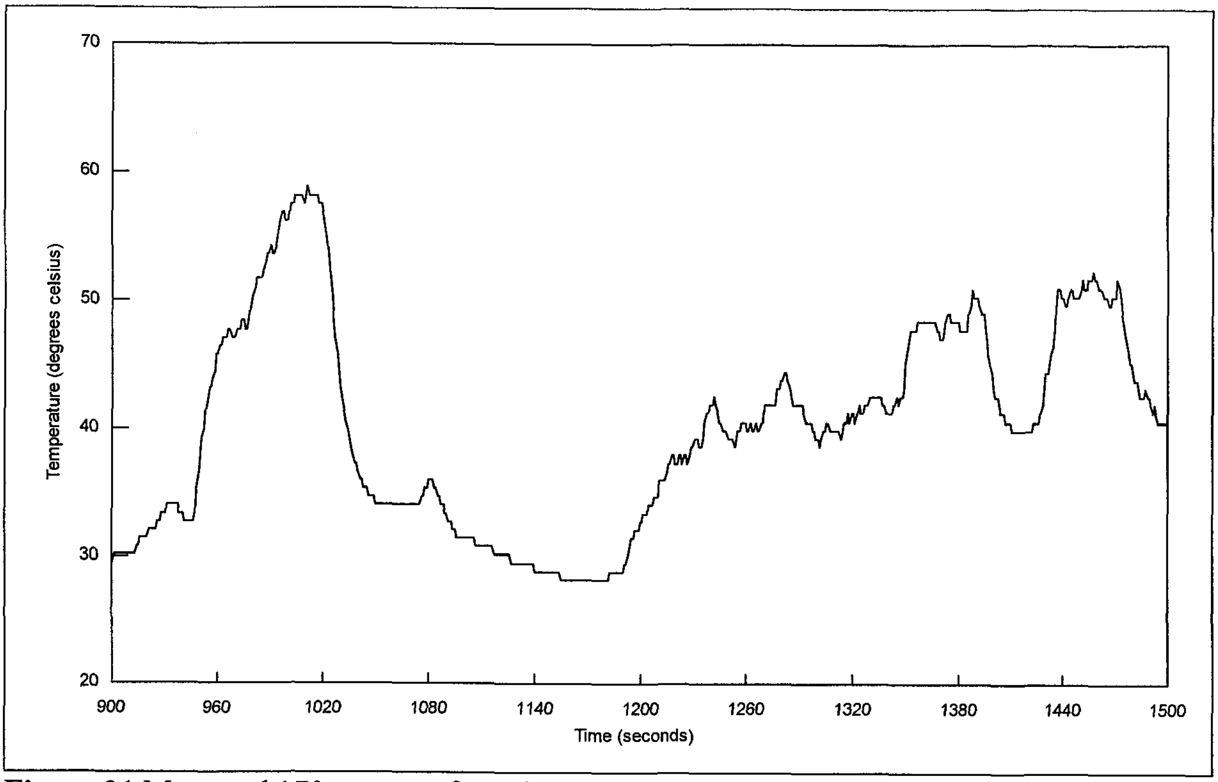

Figure 21 Measured 170 mm woofer voice coil temperature (600 s period).

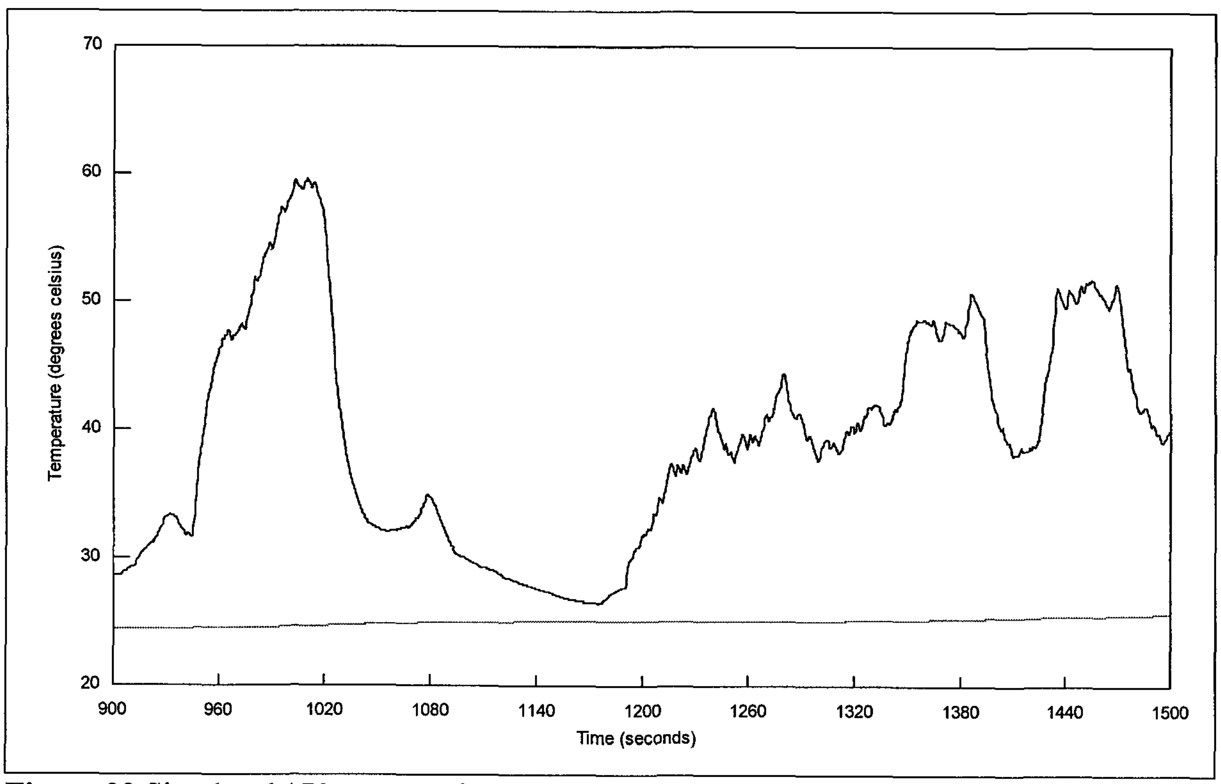

Figure 22 Simulated 170 mm woofer voice coil and magnet temperatures (600 s period).

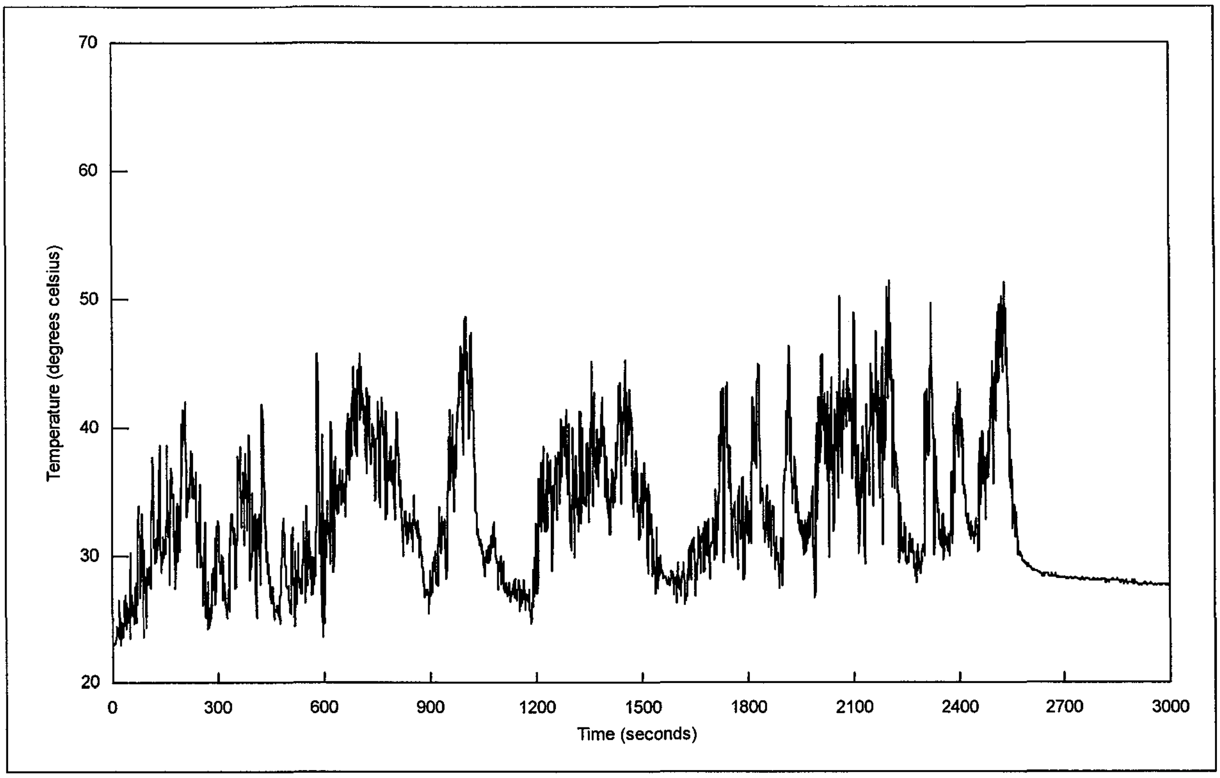

Figure 23 Measured 70 mm unit voice coil temperature (3000 s period).

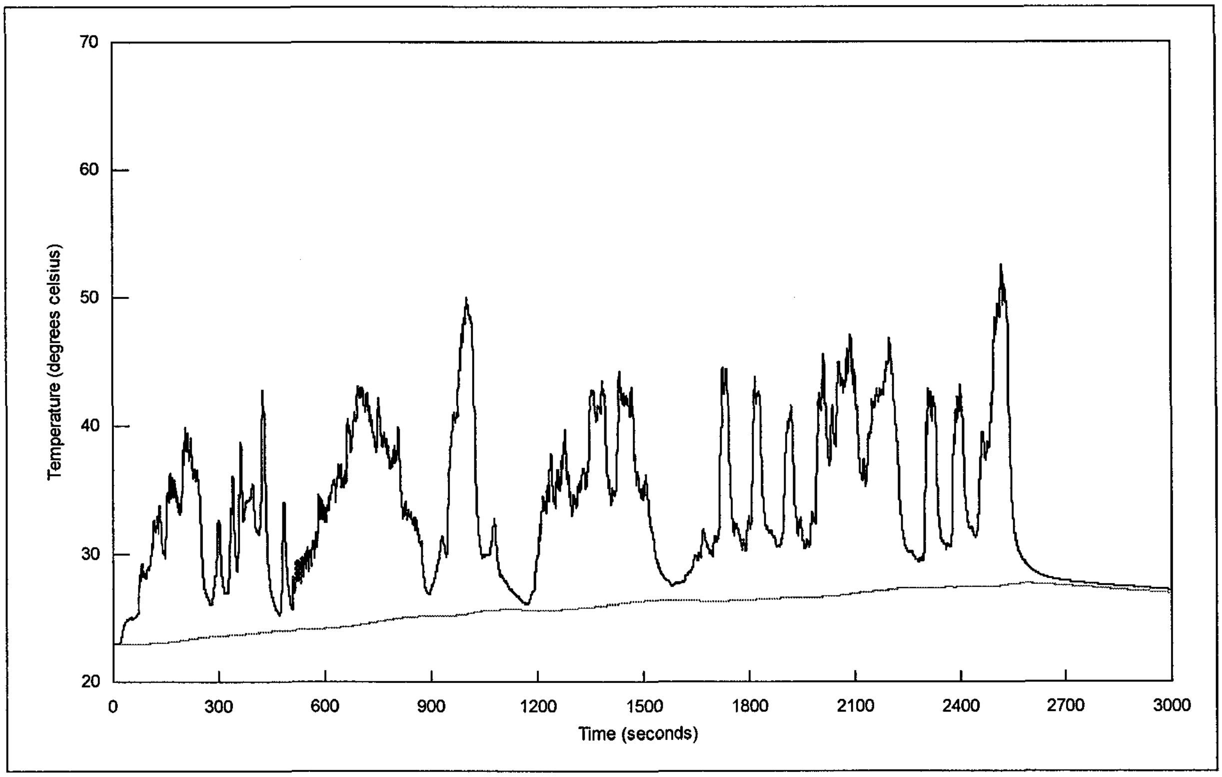

Figure 24 Simulated 70 mm unit voice coil and magnet temperatures (3000 s period).

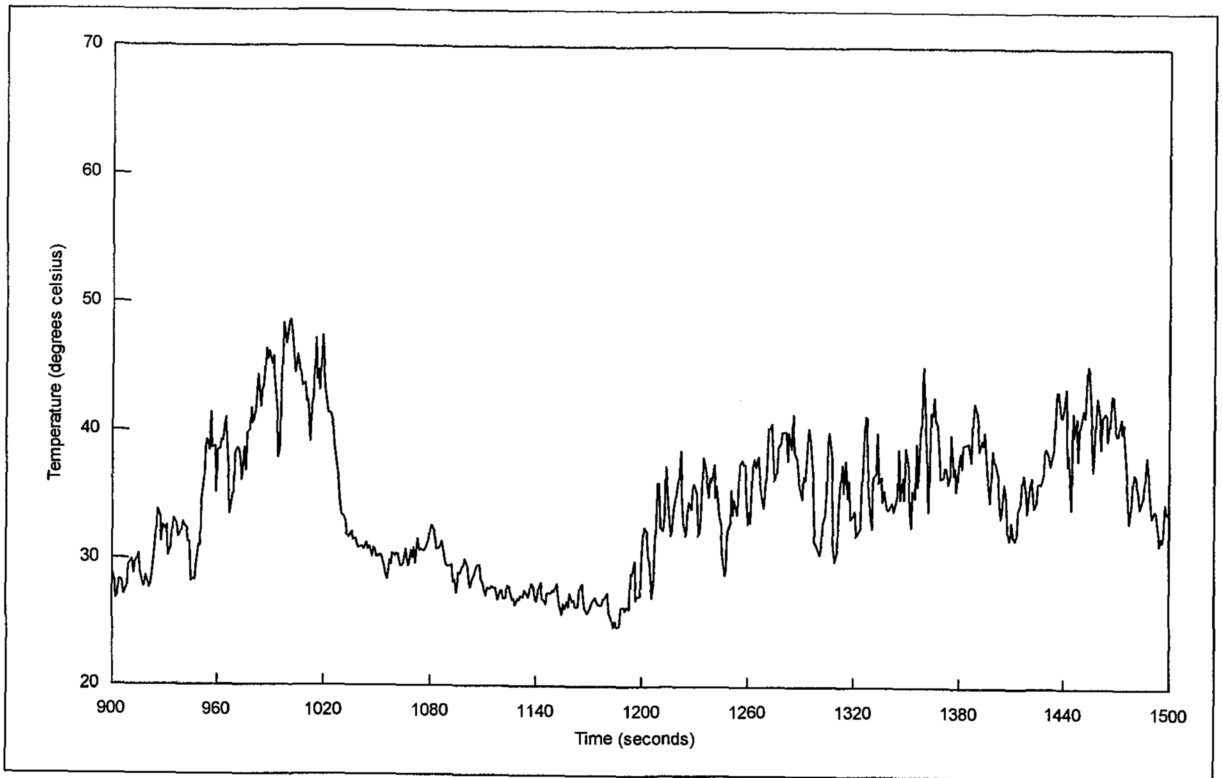

Figure 25 Measured 70 mm unit voice coil temperature (600 s period).

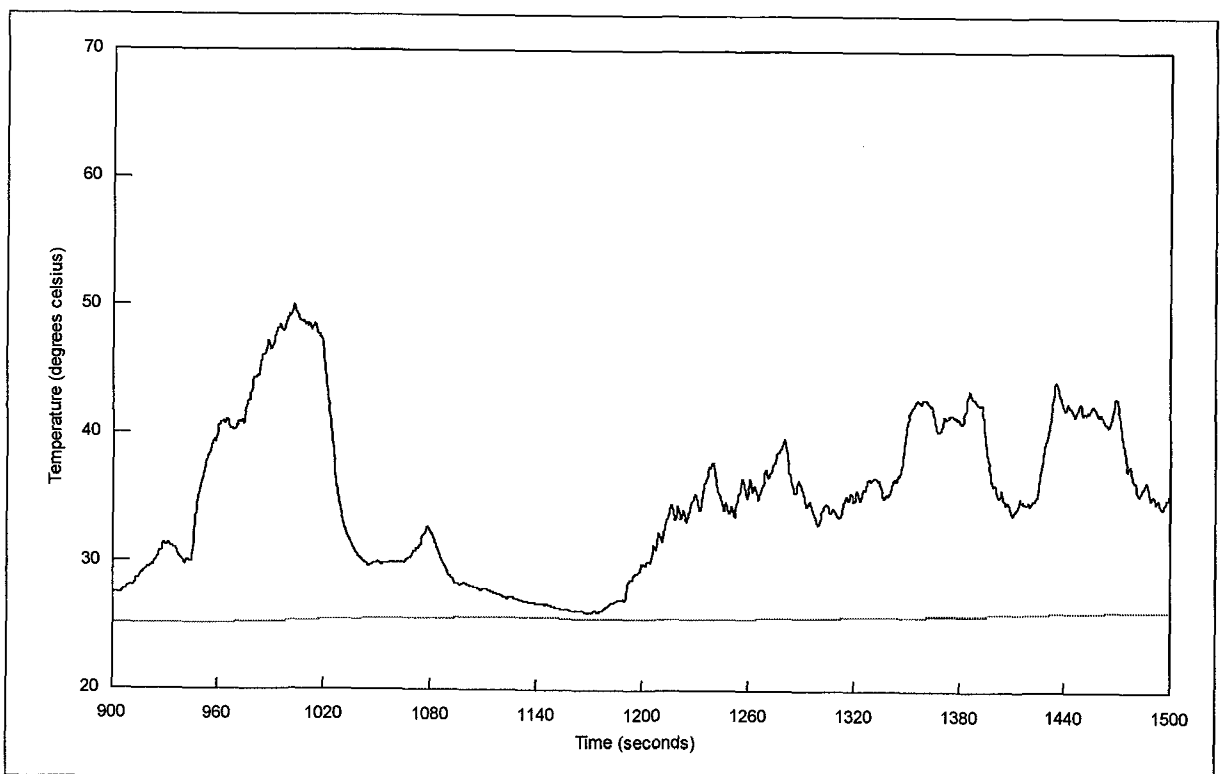

Figure 26 Simulated 70 mm unit voice coil and magnet temperatures (600 s period).

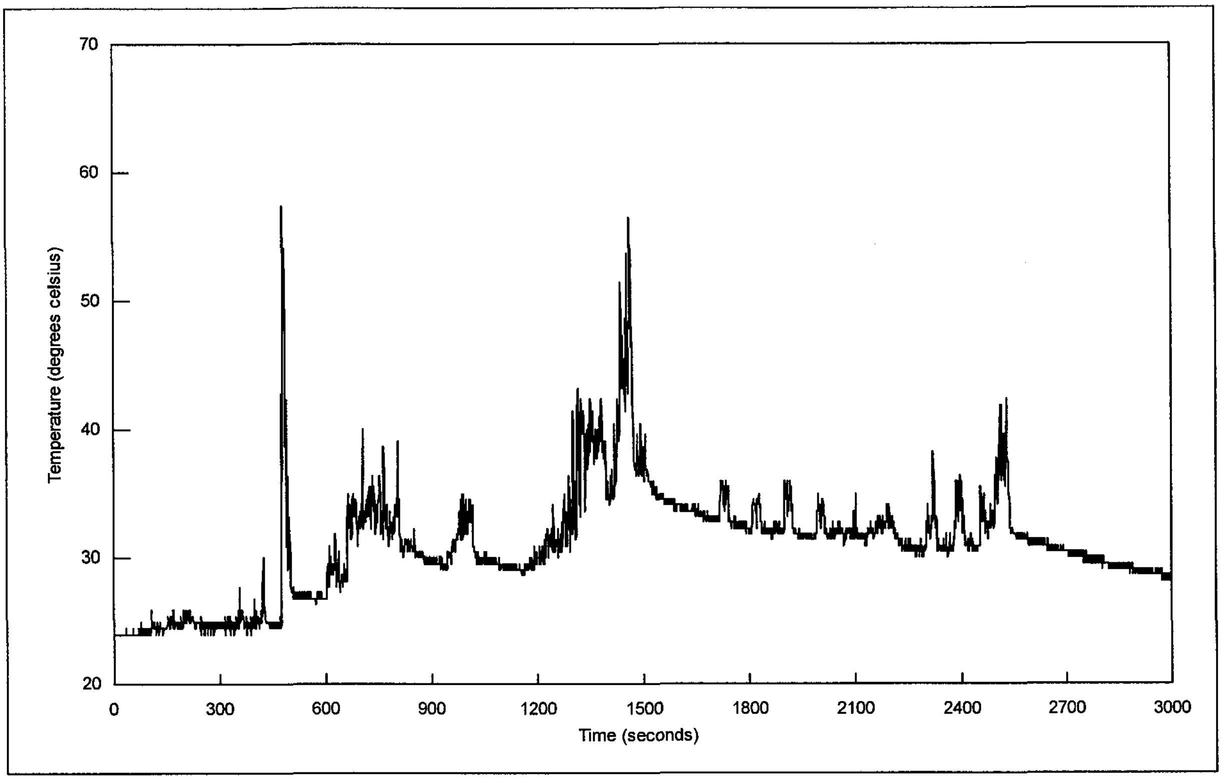

Figure 27 Measured 19 mm tweeter voice coil temperature (3000 s period).

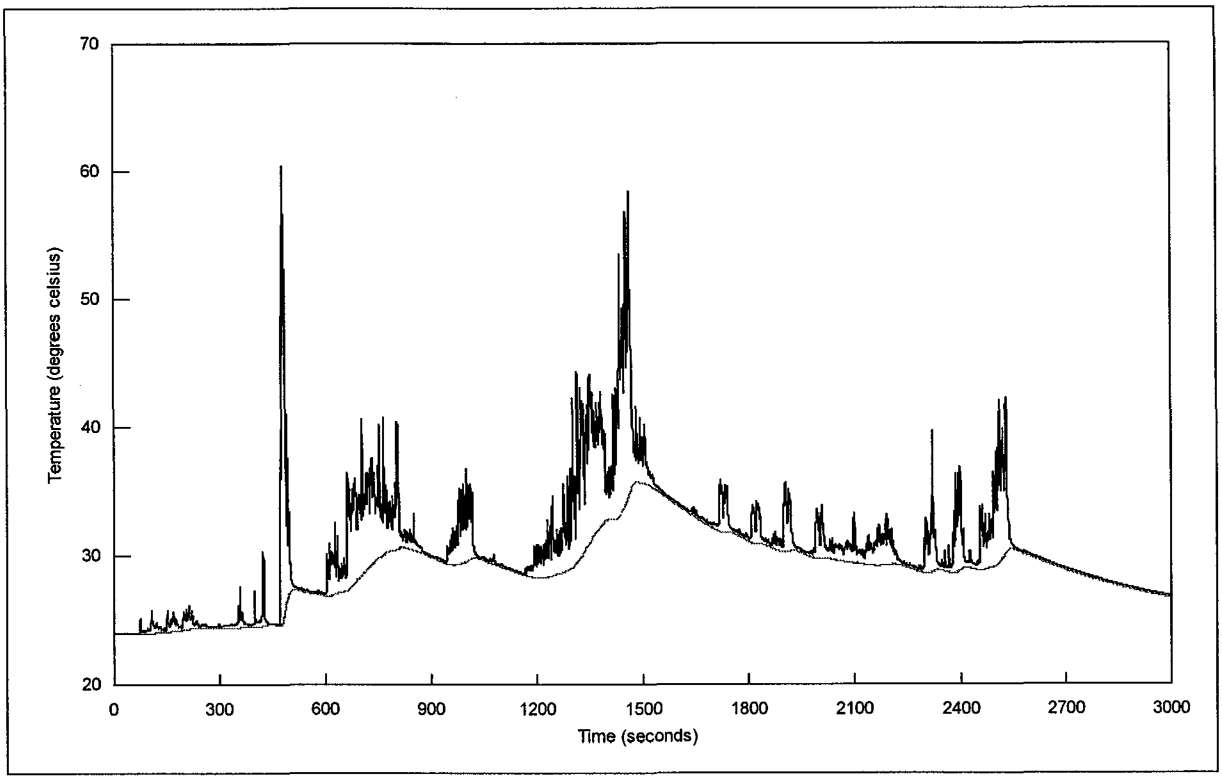

Figure 28 Simulated 19 mm tweeter voice coil and magnet temperatures (3000 s period).

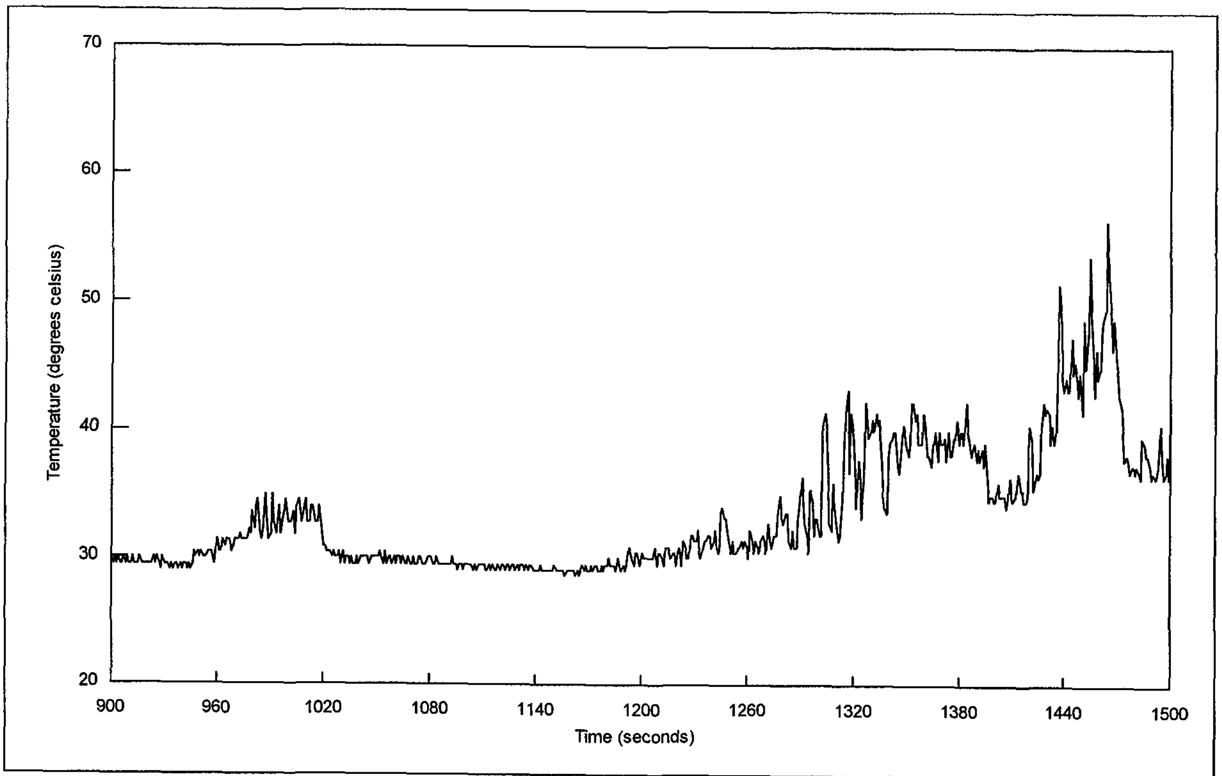

Figure 29 Measured 19 mm tweeter voice coil temperature (600 s period).

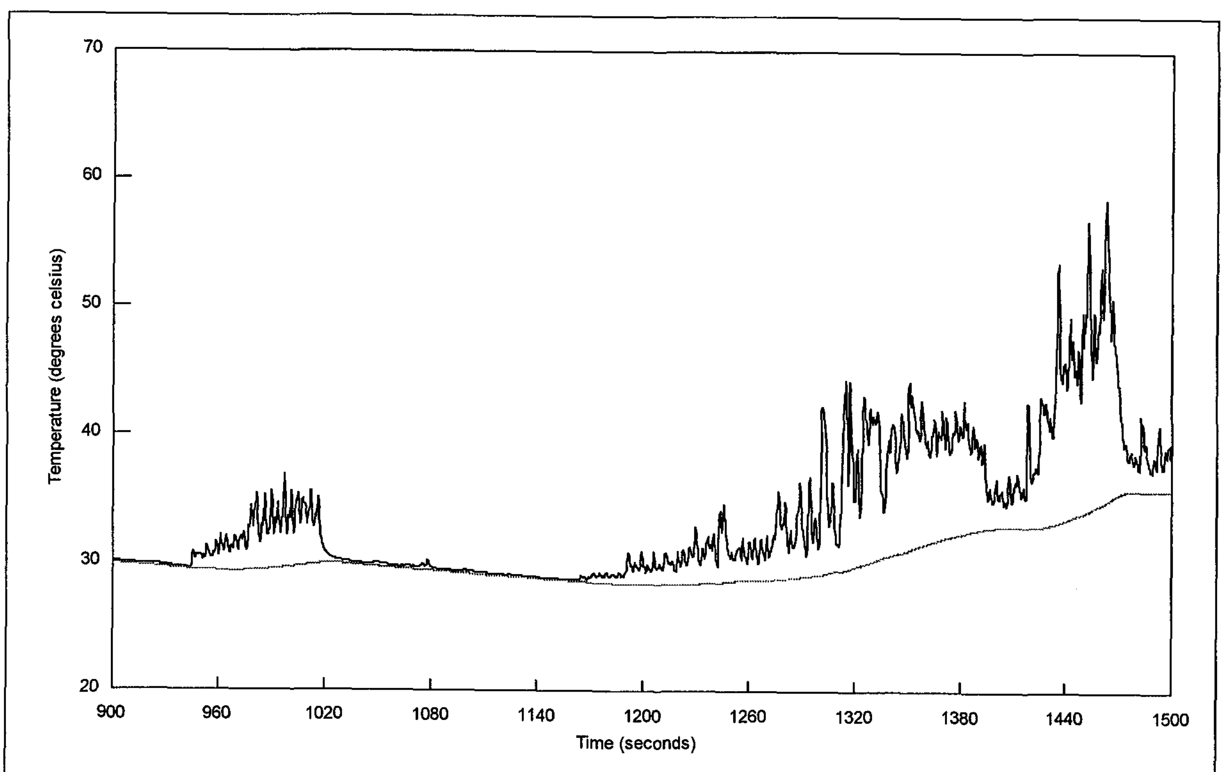

Figure 30 Simulated 19 mm tweeter voice coil and magnet temperatures (600 s period).